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by

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## ABSTRACT

Two user-oriented computer programs are presented and described. The first is suitable for handling efficiently typical analysis and design problems involving linear arrays of parallel thin-wire antennas. The second is designed to enable efficient analysis of radiation from vertical wire antennas over systems of radial ground wires. Examples are given to illustrate various applications of both programs. Special attention is devoted to use of the first program together with a standard optimization procedure to design linear arrays of wire elements with unequal spacings and/or unequal wire lengths.

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# 1. INTRODUCTION

This report has three objectives. The first is to present and describe a user-oriented computer program suitable for handling typical radiation problems involving linear arrays of parallel thin-wire antennas. This program assumes that the wires are all of the same radius, although they may be of different lengths. It is also assumed the wires are centerfed with feed points all in the same plane and that the wires are lossless with no externally applied loading. The program is based on an analysis procedure suggested by Harrington and represents an application of the method of moments. [1,2] Within Harrington's general formalism the subsectional piecewise sinusoidal functions suggested by Richmond [3] are used for both expansion and weighting resulting in a Galerkin solution to the analysis problem. The corresponding computer program presented here appears to have an advantage with respect to convergence over previous programs, [4,5] that were based on other sets of expansion and weighting functions. This appears especially true for radiation problems involving unloaded thin wires near resonant lengths. The program is described in Chapter 2 of this report and is characterized by very simple data input requirements. The program listing is included in Appendix B.

A second objective of this report is to demonstrate use of the program described above together with an appropriate optimization procedure to treat some typical optimization and design problems of interest. These include design of unequally spaced arrays to provide equal sidelobe patterns, selection of wire lengths and interelement spacings to optimize array directivity, and reduction of pattern sidelobes through adjustments of wire lengths. Results for several typical problems are included in Chapter 3 along with a discussion of the optimization procedure used.

The third objective of this report is to present a modified version of a user-oriented computer program that was presented in an earlier Scientific Report [4] together with some results of its application. This program is

another example of use of the Galerkin procedure in that subsectional piecewise linear (triangle) functions are used for both expansion and weighting. The initial program was written specifically to analyze radiation and scattering by complicated configurations of arbitrarily bent and interconnected thin wires. As yet, however, data have only been presented for certain wire-cross scattering problems [5,6] and for certain radiation problems involving rectangular and circular wire loops, double-V antennas, and symmetrical-T antennas. [5,7] The modified program presented here is useful for handling certain situations involving special symmetry such as vertical wire antennas over radial wire (counterpoise) systems. As mentioned earlier, examples of its use are included. The program is described in Chapter 4, and the listing is presented in Appendix C.

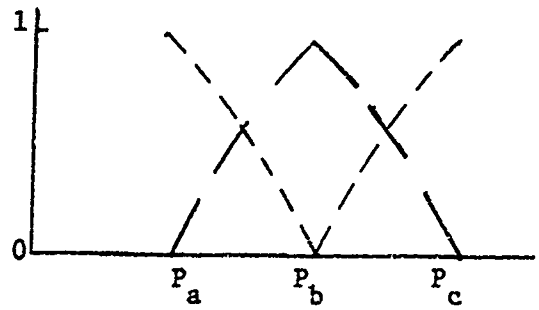
## 2. THE ANALYSIS PROGRAM

### 2.1 Summary of the Method

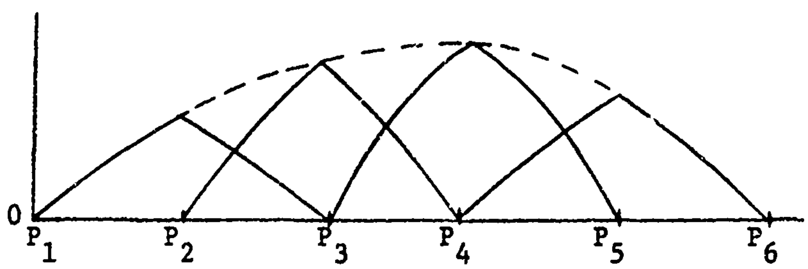
The details of Harrington's general method of analysis as applied to thin-wire antennas are readily available elsewhere. [1,2] It is assumed here the wires are thin and that current flows only in the axial direction of each. Current and charge densities are approximated by filaments of current and charge on the wire axes. The boundary condition regarding the tangential component of the electric field at the wire surfaces is satisfied (approximately) by requiring that the axial component vanish at the surface of each wire. In the subsectional approach used here each wire is thought of as divided into a number of short segments or subsections connected together. The integrodifferential equation characterizing the analysis problem is then reduced to a matrix equation of the form

$$[V] = [Z][I] \tag{1}$$

by expanding each wire current in a sequence of subsectional expansion functions where each is nonzero only over a small portion of a wire; that is, over a small number of subsections. There are many useful choices of sets of expansion functions, and many have been investigated in detail [1,2]. For the program presented here the piecewise sinusoidal functions suggested by Richmond are used, and these are depicted in Fig. 1. [3]




(a) Sinusoidal function




(b) Piecewise sinusoidal approximation

Fig. 1 - Subsectional bases and functional approximation.

In order to compute the elements of the matrices in (1) by the method of moments it is necessary to define a set of testing or weighting functions in addition to the current expansion functions used. If the same functions are used for both expansion and testing then the result is known as a Galerkin solution to the analysis problem. This is the procedure used here.

With reference to Fig. 1 the nth current expansion function can be written as (the wires of the array are all assumed to be z-directed)

$$\vec{I}_n(z) = \hat{u}_z \frac{\sin k(z-z_{n-1})}{\sin k(z_n-z_{n-1})} \qquad z_{n-1} \le z < z_n$$

$$= \hat{u}_z \frac{\sin k(z_{n+1}-z)}{\sin k(z_{n+1}-z_n)} \qquad z_n \le z < z_{n+1} \tag{2}$$

where $\hat{u}_z$ is the obvious unit vector and $k = 2\pi/\lambda$. The field corresponding to this single current function is given by [8]

$$E_\rho = \frac{j30}{\rho}\left[\frac{(z-z_{n-1})e^{-jkR_1}}{R_1 \sin k(z_n-z_{n-1})} - \frac{(z-z_n)e^{-jkR_2}\sin k(z_{n+1}-z_{n-1})}{R_2 \sin k(z_n-z_{n-1})\sin k(z_{n+1}-z_n)} + \frac{(z-z_{n+1})e^{-jkR_3}}{R_3 \sin k(z_{n+1}-z_n)}\right] \qquad (3)$$

$$E_z = j30\left[-\frac{e^{-jkR_1}}{R_1 \sin k(z_n-z_{n-1})} + \frac{e^{-jkR_2}\sin k(z_{n+1}-z_{n-1})}{R_2 \sin k(z_n-z_{n-1})\sin k(z_{n+1}-z_n)} - \frac{e^{-jkR_3}}{R_3 \sin k(z_{n+1}-z_n)}\right] \qquad (4)$$

where $E_\rho$ and $E_z$ are, of course, the field components normal to and parallel to the direction of the current element respectively. If $I_n$ is the complex amplitude of the nth current expansion function, then the total current of the array can be written as

$$\vec{I}(z) = \sum_n I_n \vec{I}_n(z) \qquad (5)$$

and the total field is an obvious corresponding generalization of (3) and (4) using (5).

Formulas for the elements of the matrices in (1) can be found easily from the general theory. [1,2,5,9] The matrix [I] is called the current matrix and is simply a column matrix of dimension equal to the total number of current expansion functions used. Each element equals the complex amplitude of the corresponding current expansion function; that is, the nth element is $I_n$. If the total number of expansion functions used is denoted by M then [Z] in (1) is a square matrix of dimension M known as

the generalized impedance matrix. If $\vec{E}_m$ is the vector field produced by the mth expansion function $\vec{I}_m(z)$ then a general formula for the typical element of [Z] denoted by $Z_{nm}$ is

$$Z_{nm} = - \int_{z_{n-1}}^{z_n} \frac{\sin k(z-z_{n-1})}{\sin k(z_n-z_{n-1})} \hat{u}_z \cdot \vec{E}_m dz + \int_{z_n}^{z_{n+1}} \frac{\sin k(z_{n+1}-z)}{\sin k(z_{n+1}-z_n)} \hat{u}_z \cdot \vec{E}_m dz \qquad (6)$$

Detailed formulas for the real and imaginary parts of $Z_{nm}$ are obtained by using (4) in (6) and are given in Appendix A of this report in terms of sine and cosine integrals. The formulas are valid if all segment lengths $(z_j-z_{j-1})$ are equal along any one wire.

The matrix [V] in (1) is a column matrix of dimension M, and its elements are related to the excitation of the array. There is an element of [V] that corresponds to each expansion function. However, it is assumed here that excitation voltages will be applied only at the center points of the array wires. Hence, the only nonzero elements of [V] are those corresponding to current expansion functions that are centered at the midpoints or feedpoints of the array wires. These nonzero elements are numerically equal to the complex excitation voltages that are applied. This result is derived directly from the general formula for [V] by using impulsive excitation functions at the points described.

Once the elements of [Z] and [V] have been computed the current distribution is found from (1) by simply inverting [Z]. Input impedances and field patterns of interest are then determined using straightforward and well-known matrix manipulations, completing the analysis problem.

## 2.2 Program Description

In the program printout of Appendix B the main program is listed first followed by the various subroutines. The example included in the Appendix is for analysis of a linear array of eight half-wave wires that are half-wavelength

spaced and all centerfed with real unit voltages. Required input data included in the main program are listed as follows:

a) The fifth statement is

AK = TP * (wire radius) (TP = $2\pi$)

which inputs the wire radius in wavelengths. (Recall that all array wires are assumed to have the same radius.) The wire radius used in the example included in Appendix B is $0.007022\lambda$.

b) The total number of wires making up the array is provided with the sixth statement of the main program. Of course, for the example considered this statement is simply N=8.

c) The eighth statement is the first of (2N-1) consecutive data statements providing information on the geometry of the array. The first N of these are typified by Y(j) = (half-length of the jth wire in wavelengths) * TP where TP is simply $2\pi$ as determined in statement four. The half-lengths of the wires included in the example of Appendix B are all $0.25\lambda$. The next (N-1) data statements indicate the positions of the array wires relative to the first. An example of these is

Y(k) = (distance of the kth wire from the first in wavelengths) * TP

Note that the wires in the array of the example are all spaced one half-wavelength apart.

d) The purpose of th DO LOOP formed by statements 23-25 of the main program is to provide information concerning excitation of the array. As mentioned earlier it is assumed the wires are all centerfed so that [V] is a column matrix with each nonzero element equalling the complex excitation in volts applied at the midpoint of the corresponding wire. In the illustrative example of the Appendix all wires are centerfed with real unit voltages.

e) Part of the computed output is the field pattern of the array taken in the principal H-plane. This pattern is computed over 180° of the azimuth angle centered on the broadside direction. Values are calculated at intervals

determined by the 26th statement of the main program. The statement MQ=3 shown in the sample printout means that intervals of three degrees are desired. If it is only necessary to compute the pattern at five degree intervals of the azimuth angle then, of course, this statement should read MQ=5.

f) The only other information required is concerned with the number of current expansion functions that should be used to represent the current for each array wire. This decision is made by way of statement 40 included in Subroutine ASCTFD. For analysis problems where an accurate representation of the current is needed to determine correctly input impedances and other quantities of interest the statement

MM = INT(Y(I) - 0.571) + 3

is recommended, where Y(I) is defined in paragraph (c) above. The total number of expansion functions for the Ith wire is [2(MM)-1] so that the relationship between this number and the wire length L is approximately

$$0 < L < \frac{\lambda}{2} \; . \; . \; . \; 2(MM)-1 = 5$$

$$\frac{\lambda}{2} < L < 0.82\lambda \; . \; . \; . \; 2(MM)-1 = 7$$

$$0.82\lambda < L < 1.14\lambda \; . \; . \; . \; 2(MM)-1 = 9$$

and so on.

This seems to be a reasonable choice based on the discussion of convergence included in Section 2.3 of this report. If, on the other hand, the primary concern of the user is with pattern characteristics as is the case with the optimization problems discussed in Chapter 3 then fewer current expansion functions are needed for each wire and the statement

MM = INT(Y(I) - 0.571) + 1

can be used. With this choice the numbers of expansion functions used for the intervals defined above are 1,3,5,... . Obviously the program user can make either of these choices or an independent one, depending on the particular kinds of results sought.

Normal printed output of the program includes all input data, the current distribution as expressed by the real and imaginary parts of the complex amplitude of each current expansion function, input impedances corresponding to each feed point, and the normalized electric-field pattern in the principal H-plane together with the normalization constant.

Program operation proceeds roughly as follows: The generalized impedance matrix [Z] is computed using instructions 53-97 with the obvious symmetry about the feed points taken into account in order to reduce overall computation time and storage requirements. This matrix is inverted using Subroutine CSMIN, a complex inversion routine authored at the University of Illinois. The current matrix [I] is determined by forming the matrix product

$$[I] = [Z]^{-1} [V] \tag{7}$$

in Subroutine MULTPY. Finally, the electric-field pattern is computed for the principal H-plane using instructions 101-120, and the input impedances corresponding to feed points are calculated using the DO LOOP of statements 146-152. The FUNCTIONS ZMN and ZM1 compute mutual terms of [Z] when the segment lengths of the corresponding wires are unequal and equal respectively. This is done with the aid of Subroutine SICI made available through the IBM Scientific Subroutine Package for the purpose of computing the required sine and cosine integrals $S_i(x)$ and $C_i(x)$. The program described here was developed with an IBM 370/155 computer and storage-time requirements are quoted with the various examples as they are presented in later sections of this report.

## 2.3 Comments on the Number of Expansion Functions

The number of current expansion functions that should be used in any given situation has been an open subject of discussion for some time. It is clear that requirements vary considerably in this respect depending on the kinds of results sought. For example, for some pattern synthesis and optimization problems it may be possible to use as few as say five expansion functions per wavelength, while in some other situations where careful analyses of current distributions are required it is advisable to use considerably more,

say 10-15 expansion functions per wavelength. This, of course, is the reason for incorporating the special instruction discussed in Section 2f above.

Figures 2-5 illustrate the kinds of results that are obtained as the numbers of expansion functions are varied for certain, single-wire problems. First, consider a half-wave centerfed wire of radius 0.007022λ. The input admittance is calculated for this problem using different numbers of current expansion functions and for two different choices of subsectional expansion functions. One of these choices involves the piecewise sinusoidal functions used here while the other involves the piecewise linear (triangle) functions used for the analysis programs developed earlier to handle arbitrary configurations of bent and interconnected wires. [4] Computed results are shown in Fig. 2 along with the corresponding experimental data reported by Mack. [10] Similar results are shown in Figs. 3-5 for longer wires. These and other available data indicate that the piecewise sinusoidal functions offer somewhat faster convergence than the functions used with the arbitrary wire program and also that at least 9 or 10 expansion functions are needed per wavelength to describe accurately the current distributions encountered in the kinds of problems considered here. As indicated by Fig. 2 an even larger number may be advisable for wires near resonant lengths.

Figure 6 provides computed information on the self and mutual admittances of two, half-wavelength, centerfed parallel wires that are spaced various distances apart and are both of radius 0.007022λ. As before, data are recorded as functions of the number of expansion functions used for each wire and also versus the separation distance d, for both the piecewise sinusoidal and the piecewise linear cases. Once again, Mack's corresponding experimental data are included. As before, the data indicate that the piecewise sinusoidal functions offer relatively rapid convergence and that approximately 10 expansion functions are needed per wavelength of wire to obtain good results for the current distributions and input admittances.

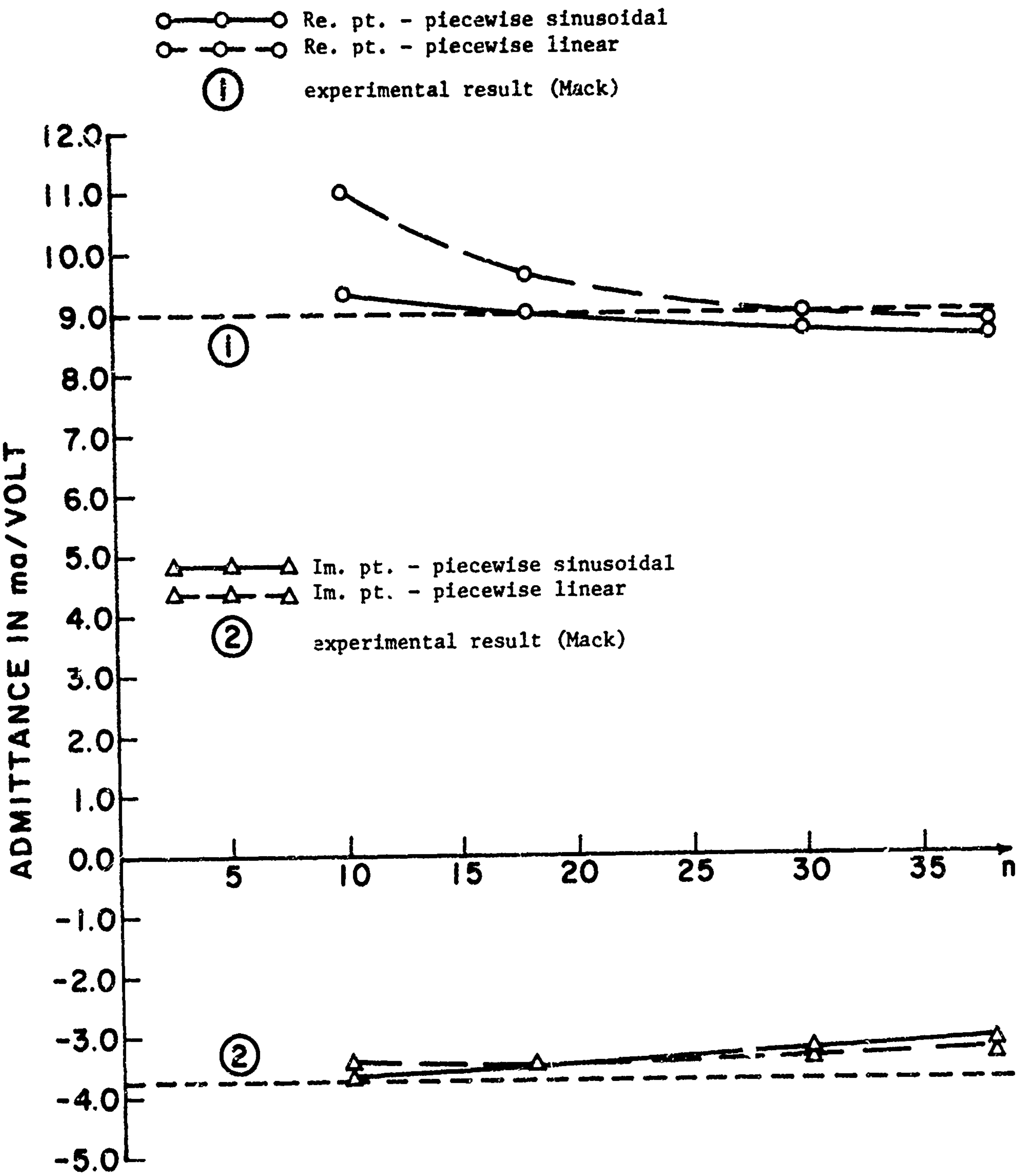



Fig. 2 - Input admittance of a centerfed linear antenna of length 0.5λ and radius 0.007022λ plotted vs. n, the number of subsectional expansion functions used per wavelength.

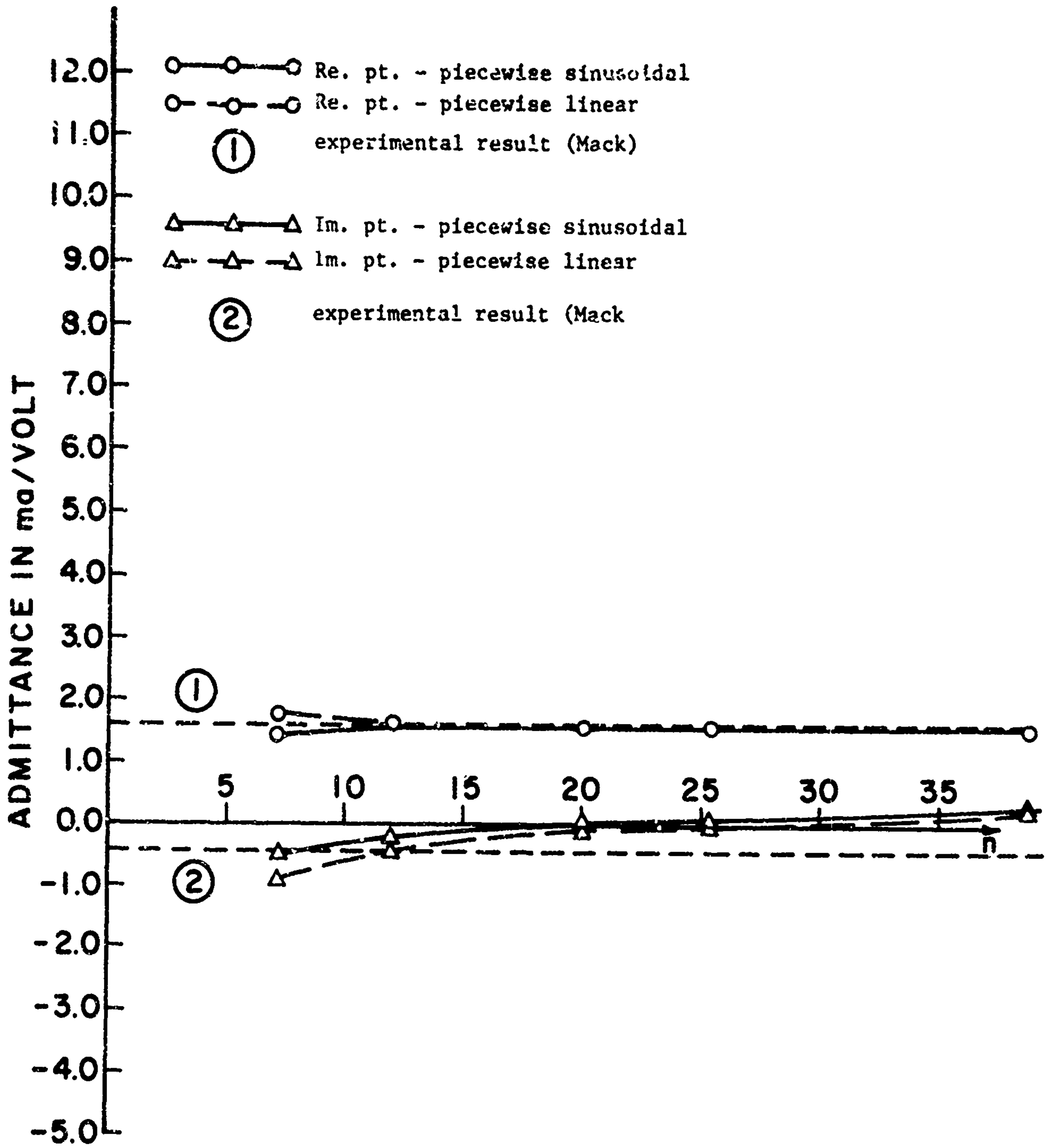



Fig. 3 - Input admittance of a centerfed linear antenna of length 0.75λ and radius 0.007022λ plotted vs. n, the number of subsectional expansion functions used per wavelength.

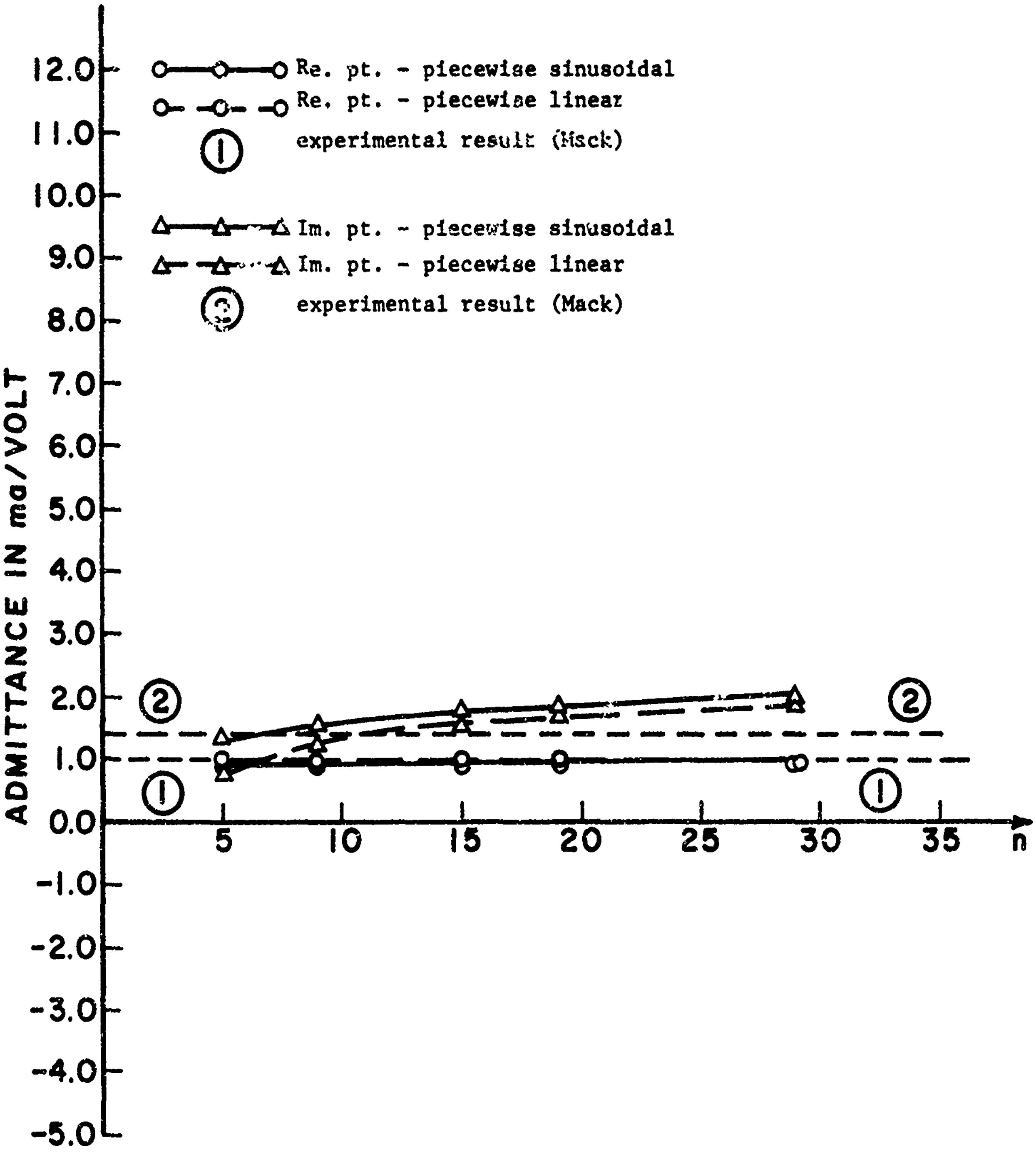



Fig. 4 - Input admittance of a centerfed linear antenna of length λ and radius 0.007022λ plotted vs. n, the number of subsectional expansion functions used for wavelength.

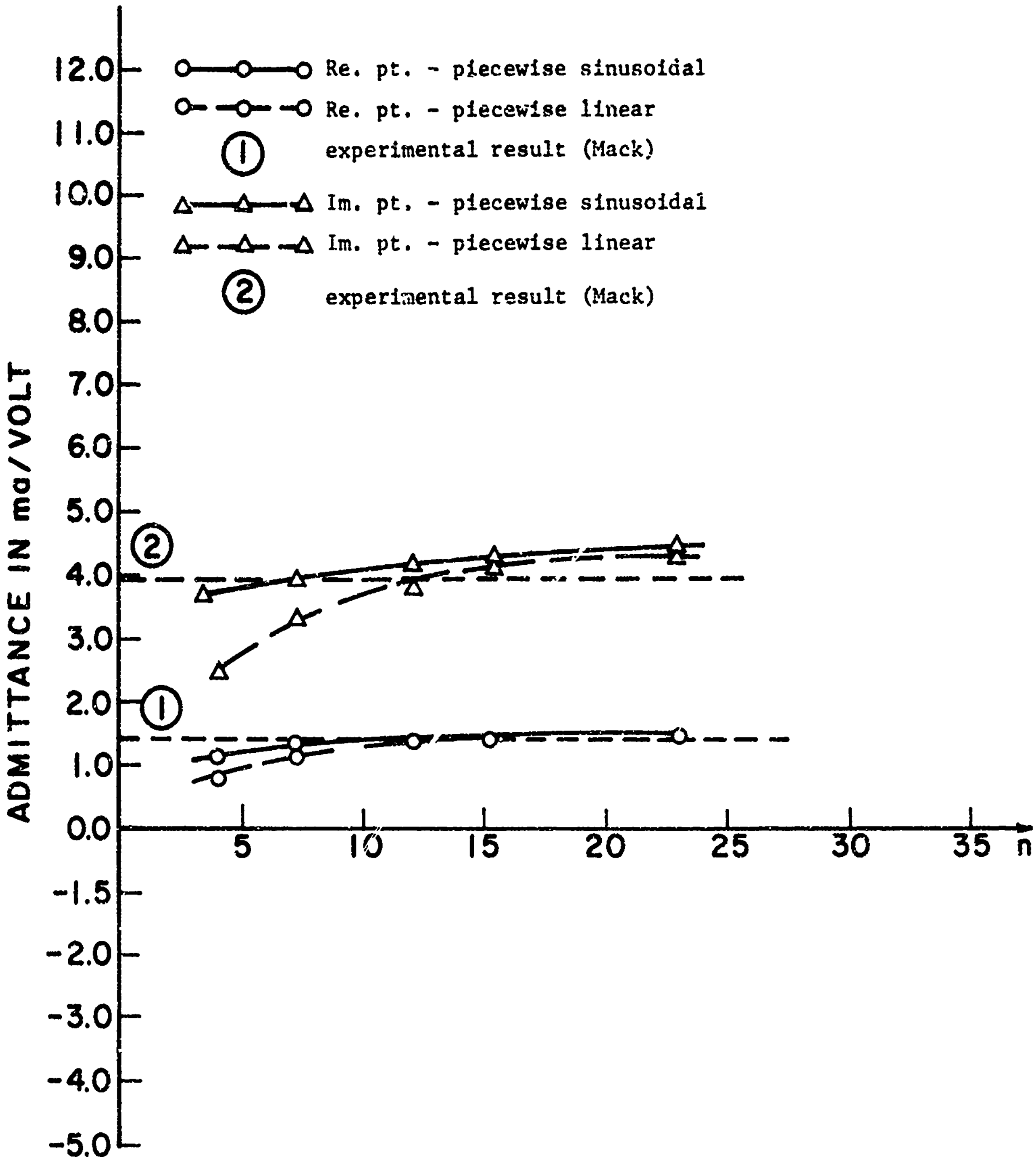



Fig. 5 - Input impedance of a centerfed linear antenna of length 1.25λ and radius 0.007022λ plotted vs. n, the number of subsectional expansion functions used per wavelength.

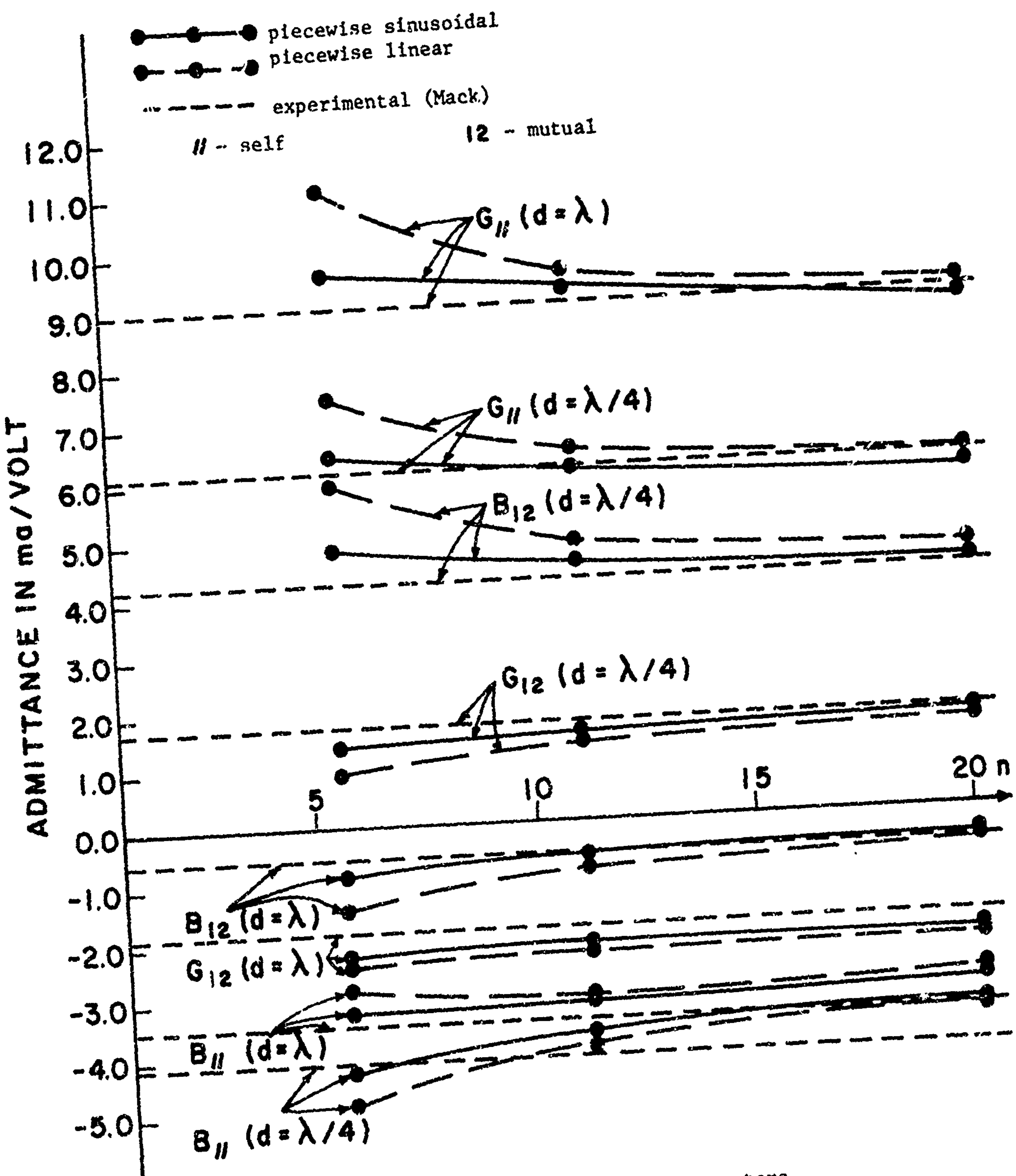



Fig. 6 - Convergence of terminal admittance parameters.

The somewhat faster convergence that characterizes the piecewise sinusoidal current approximation for problems involving thin wires is due in part to the inherent shape of the individual expansion functions used and also in part to the fact that the integrations required to obtain the elements of [Z] can be performed analytically. This is in contrast to certain programs based on the piecewise linear current approximation that incorporate a substantial number of approximate numerical integrations. The piecewise linear data of Figs. 2-6 were derived using one of these. It is interesting to note that convergence is significantly improved even in the piecewise linear case if the integrations required to obtain [Z] are performed analytically [11].

Another advantage of the formulation used here is that there is no tendency towards matrix instability as the number of expansion functions is increased, a difficulty encountered with some other methods available for handling thin-wire problems.

## 3. OPTIMIZATION PROBLEMS

### 3.1 Introduction

In this chapter an optimization procedure is used together with the analysis program described in the previous chapter to treat certain optimization and design problems involving arrays of parallel thin-wire antennas. As mentioned earlier these problems include design of unequally spaced arrays to provide equal-sidelobe patterns, reduction of pattern sidelobes through adjustments of wire lengths, and selection of wire lengths and spacings to optimize array directivity. The optimization procedure used is due to Rosenbrock [12] and represents a modified version of the method of steepest descent. This method does not require calculation of any derivatives and is particularly well-suited to automatic calculation by digital computer. The method has been used by other antenna engineers in treating challenging optimization problems of practical interest.

### 3.2 Design of Unequally Spaced Arrays

The analysis program presented in the previous chapter has been used with the optimization method of Rosenbrock [12] to develop a design technique for linear arrays of unequally spaced, parallel-wire antennas. As before the wires are assumed to be centerfed and all of the same radius. In this case it is also assumed that each wire is excited with a real unit voltage, and that the interelement spacings are symmetrical about the array center.

During the course of this investigation several error criteria were tried in association with the optimization procedure used. Most of these resulted in relatively poor or slow convergence to acceptable solutions, and quite often an acceptable solution was never reached within the reasonable time constraints placed on the computer. Of course, the error criterion is the quantity to be minimized as the optimization procedure progresses and the most successful of those tried is given by

$$\varepsilon = \sum_{i=2}^{p} \left| |T(1)|^2 - |T(i)|^2 \right| \tag{8}$$

where T(i) is the peak of the ith sidelobe with T(1) being the peak of the sidelobe closest to the main beam. (Note that the resulting broadside pattern is symmetrical about the main beam since the excitations are uniform and the spacings are symmetrical about the array center.) The limit p is simply the number of sidelobes to be controlled on either side of the main beam. The starting point for the iterative procedure is the pattern of an equally spaced uniform array with a given number of elements. The result is an equally excited, unequally spaced array with nearly equal sidelobes. Of course the design parameters are the interelement spacings, although the spacing with respect to the array center of the outermost symmetrical pair of elements is kept fixed at its initial value so that the beamwidth will remain essentially unchanged from its value at the start.

As an example consider an 8-element linear array of centerfed, half-wave, parallel wires that are half-wavelength spaced and all of radius

0.007022λ. With equal in-phase excitations the program of the previous chapter can be used easily to obtain the current distributions, input impedances, and the H-plane pattern of Fig. 7. The program can then be used in conjunction with the optimization procedure and error criterion given by (8) to obtain a new pattern with reduced and nearly equal sidelobes using only the interelement spacings as design parameters. The positions of the elements of the outermost pair are kept fixed to provide roughly the beamwidth of the uniform array and the remaining three interelement spacings are adjusted to achieve nearly equal sidelobes at a reduced level. The resulting pattern is also shown in Fig. 7 along with the initial and final spacing configurations. Note that because of symmetry the spacings are shown for only half the array and the patterns are shown for only one quadrant.

Figures 8-10 show results of applying the same procedures to linear arrays of 9, 12, and 18 parallel, half-wave wires respectively. It is noted that there is no substantial beamwidth deterioration even though the sidelobes have been reduced in overall level. These computations were all performed using an IBM 370/155 computer with total times of 25, 36, 70, and 185 seconds required for obtaining the results shown in Figs. 7-10, respectively.

Finally, it should be noted that a different error criterion was used earlier in conjunction with an alternative analysis procedure and an optimization method due to Fletcher and Powell [13] to treat the same problem discussed above [14]. However, the rate of convergence proved to be a serious problem in that case and the present choices appear to be much more satisfactory for this particular problem.

### 3.3 Use of Unequal Wires Lengths

A second problem treated successfully using the optimization method of the previous section involves use of array wire lengths rather than the interelement spacings as design parameters. The objective is to reduce the sidelobes of an equally excited, equally spaced array of (initially) equal-length

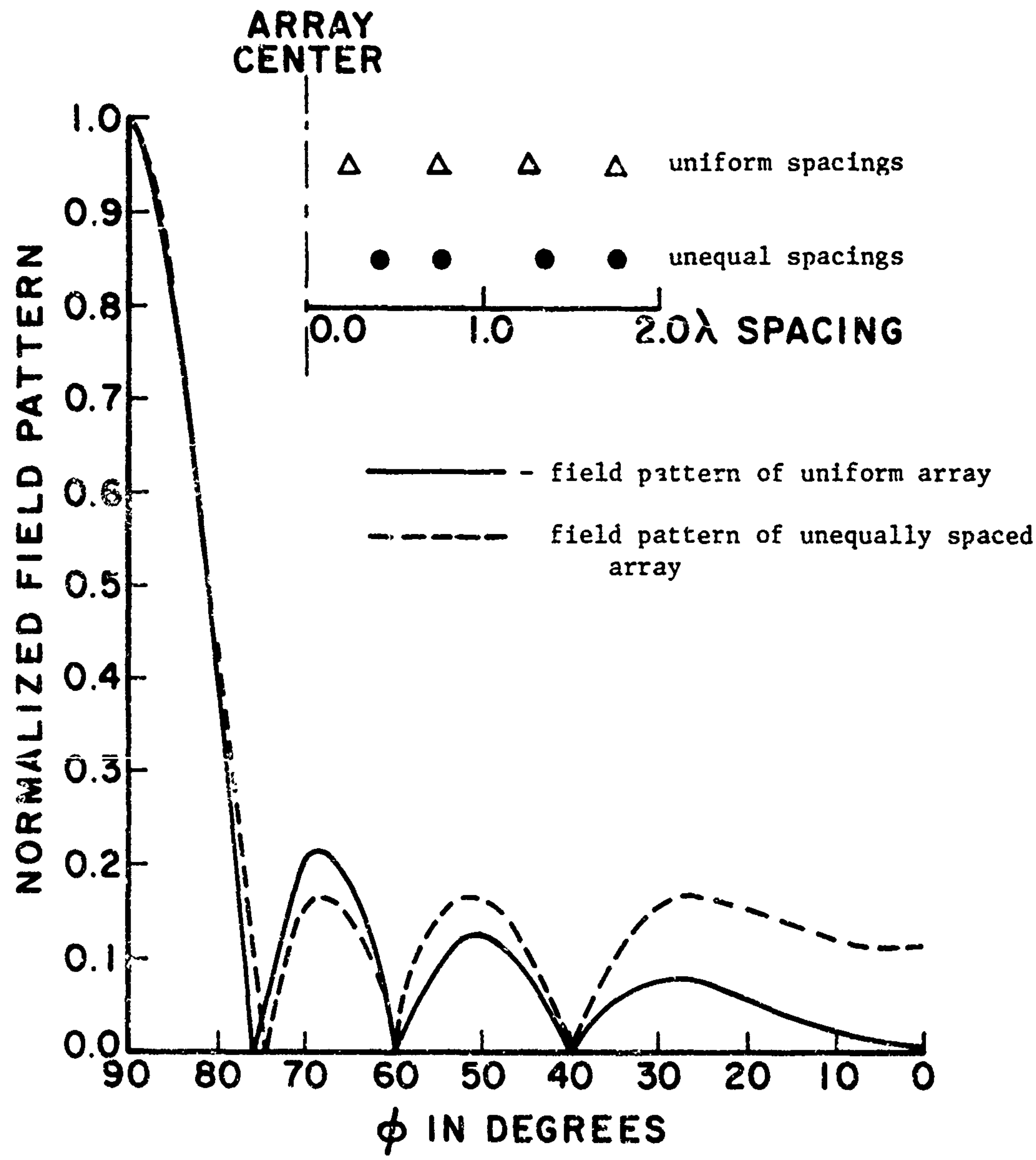



Fig. 7 - Field patterns of an 8-element uniform array along with the pattern of a similar array with unequal spacings designed for equal sidelobes.

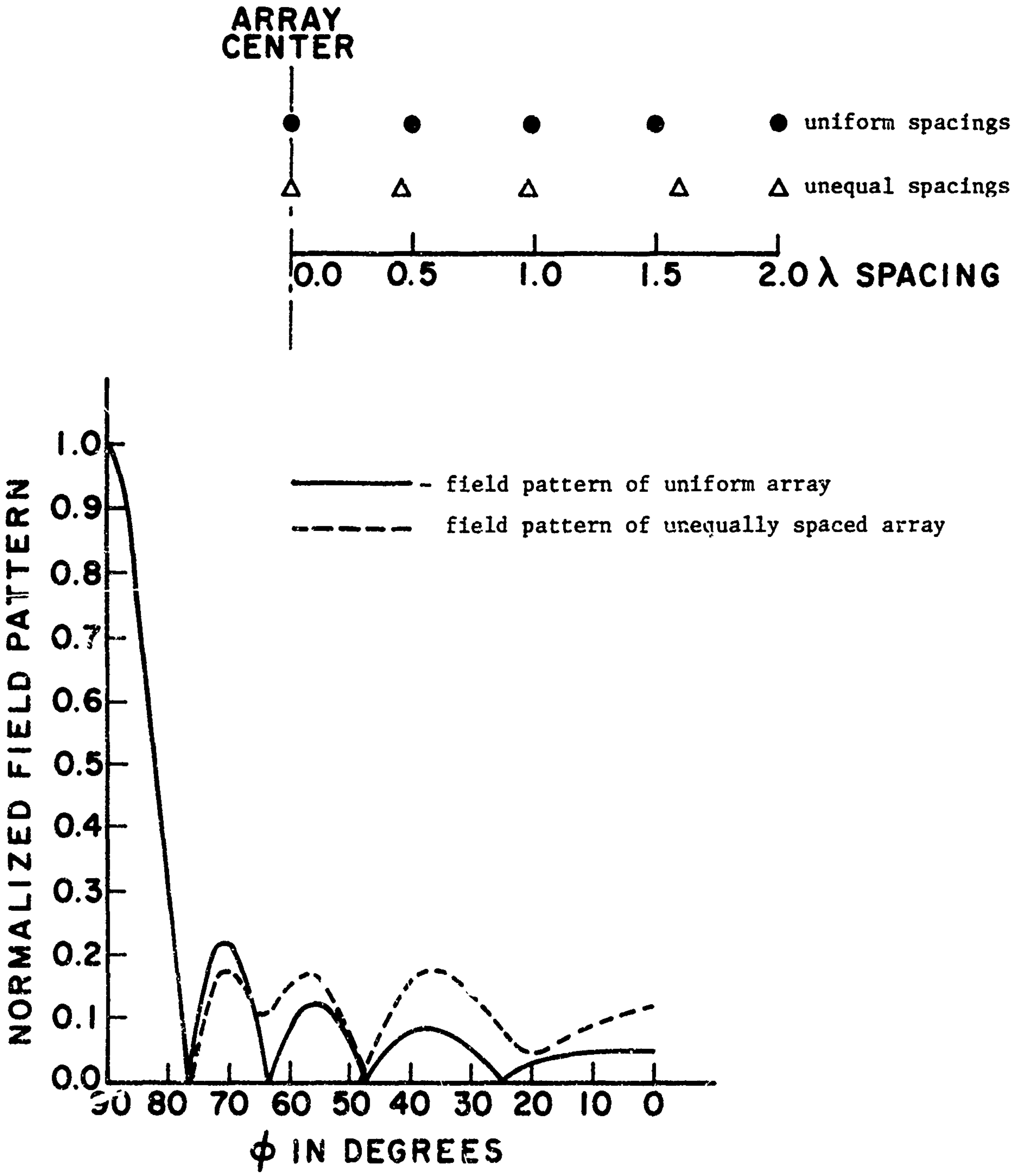



Fig. 8 - Field pattern of a 9-element uniform array along with the pattern of a similar array with unequal spacings designed for equal sidelobes.

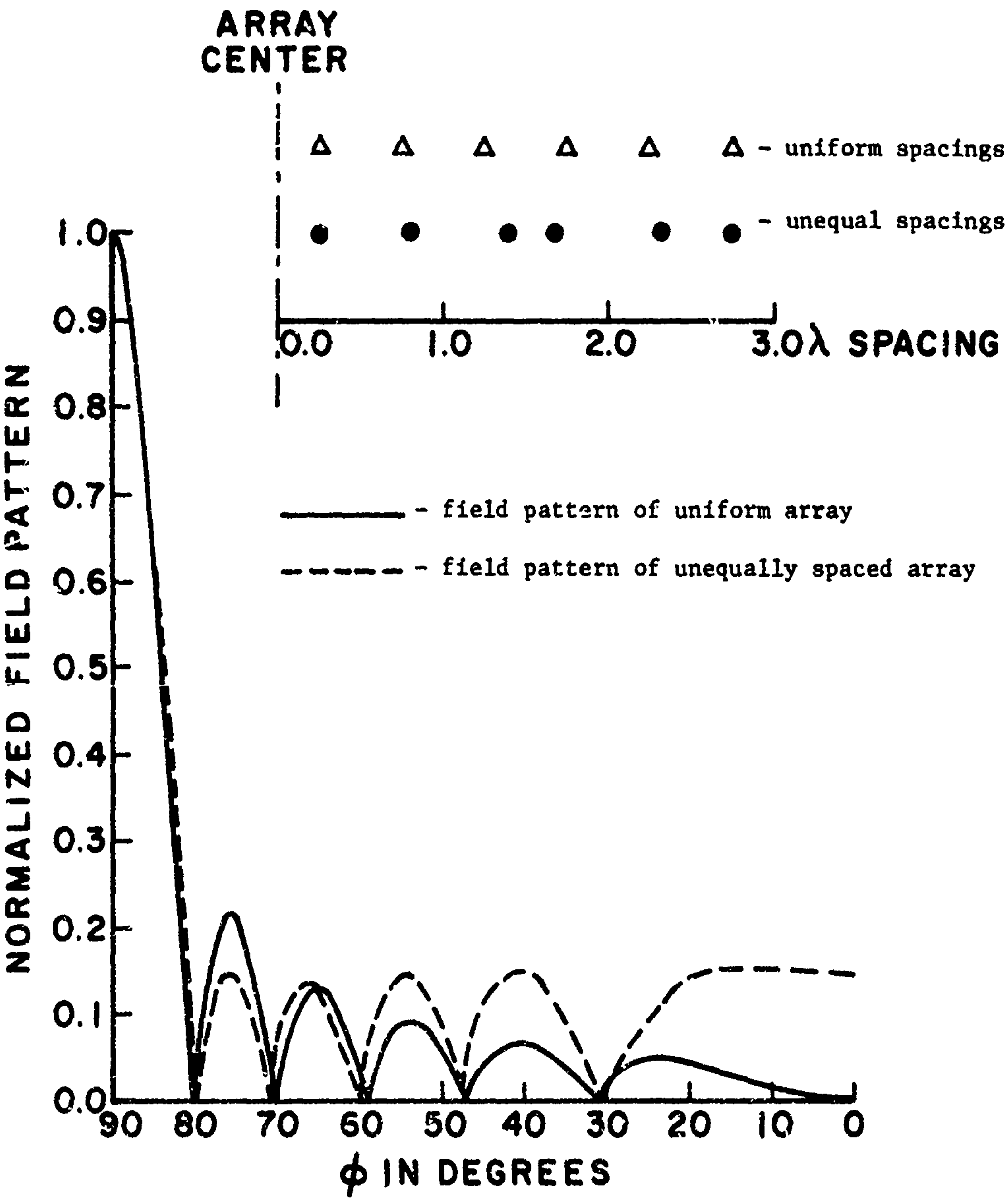



Fig. 9 - Field pattern of a 12-element uniform array along with the pattern of a similar array with unequal spacings designed for equal sidelobes.

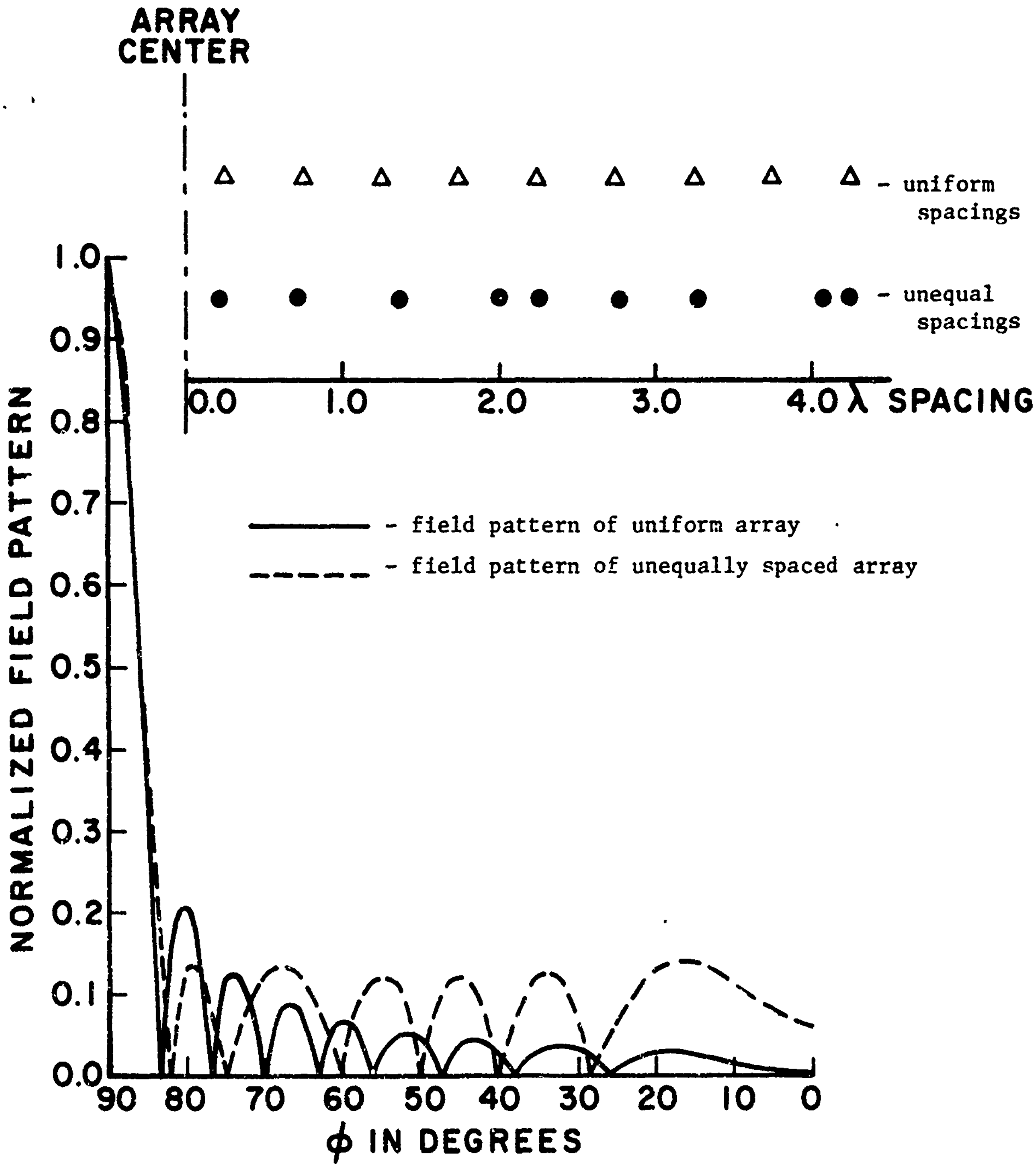



Fig. 10 - Field pattern of an 18-element uniform array along with the pattern of a similar array with unequal spacings designed for equal sidelobes.

wires by adjusting the wire lengths. As before, it is assumed here that the resulting array remains symmetrical about its center and also with respect to the plane containing the feed points. The wires are again assumed to be centerfed and all of the same radius.

In this case the most success was achieved using an error criterion given by

$$\varepsilon_1 = \sum_{i=1}^{n} \left| |T(i)|^2 - |T_o|^2 \right| \quad \text{for} \quad |T(i)| > |T_o|$$

$$= 0 \qquad \text{for} \quad |T(i)| \leq |T_o| \tag{9}$$

where in this case T(i) is the value of the field pattern in the ith specified direction and n is the number of observations calculated (e.g. one every two or three degrees) over the sidelobe region of the pattern. The quantity $|T_o|^2$ is simply the square of the desired sidelobe level. The starting point for the iterative procedure is the pattern of an equally spaced uniform array with a given number of equal-length wires. The result is an equally excited, equally spaced array of wires of different lengths with sidelobes near the desired level.

As an example consider a 12-element array of centerfed, parallel wires that are half-wavelength spaced, uniformly excited, and initially all one half-wavelength long. The wires are all of radius 0.007022λ, and the initial H-plane pattern is shown in Fig. 9. The analysis program of the previous chapter can be used together with the optimization procedure and error criterion given by (9) to obtain the new pattern of Fig. 11 by adjusting only the wire lengths. The desired level in this case was $\pm 0.1$ corresponding to $|T_o|^2 = 0.01$. This corresponds to a desired sidelobe level of -20 dB. Figure 11 also includes the final wire lengths which are assumed symmetric about the array center.

Results for this example are quite close to those desired as indicated by Fig. 11. However, the procedure is time-consuming with convergence much

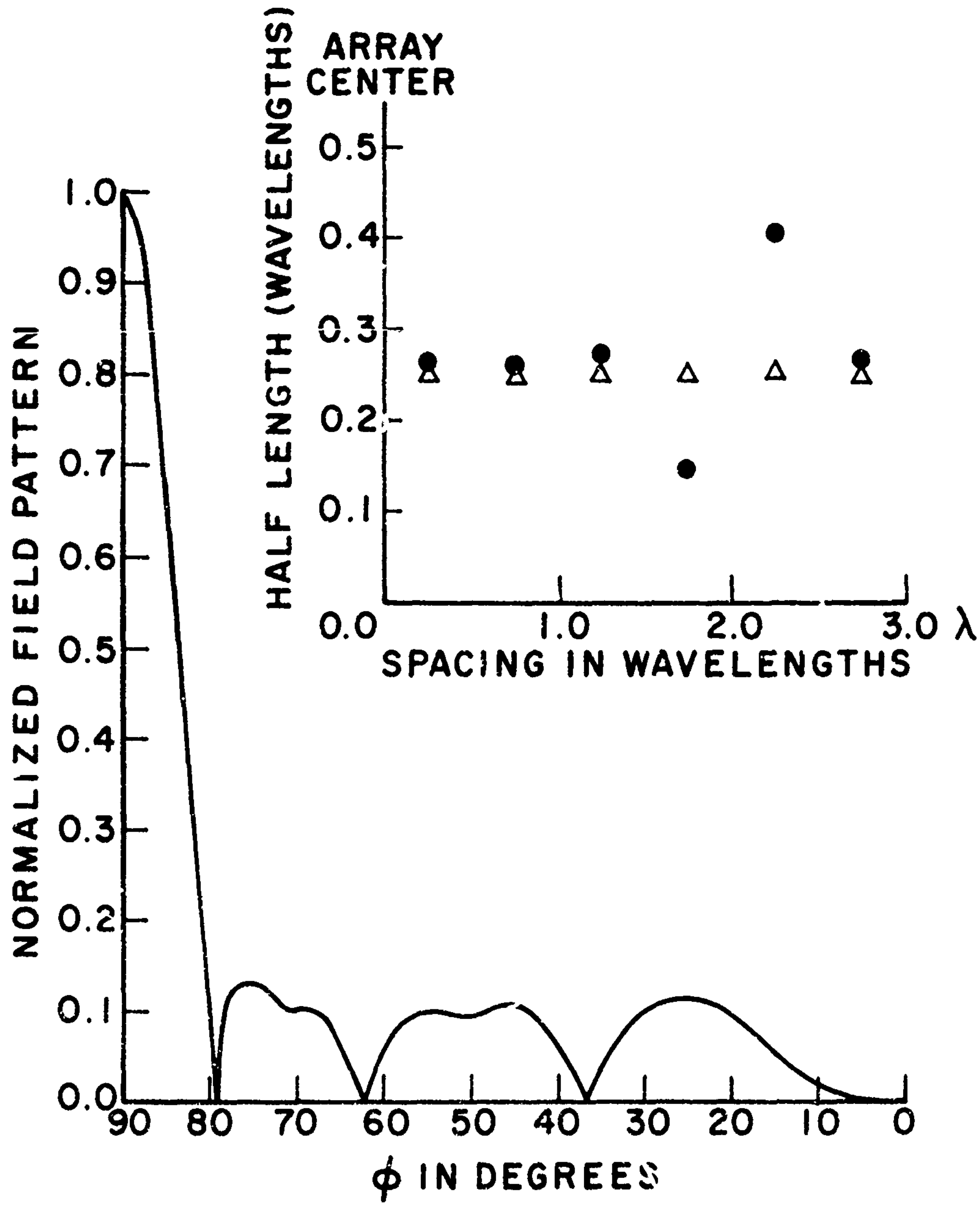



Fig. 11 - Pattern of a 12-element array with sidelobe minimized with .espect to wire lengths.

slower than in the examples of the previous section. A total of about nine minutes was required to obtain the data corresponding to Fig. 11 using an IBM 370/155 computer.

### 3.4 Adjustments of Both Lengths and Spacings

An obvious extension of the work for the last two sections involve adjustments of both wire lengths and interelement spacings simultaneously to achieve reduced sidelobes. Figure 12 shows the final result of minimizing the error criterion given by (9) for an 8-element array of parallel wires. The starting point for the iterative procedure was the initial pattern of Fig. 7 corresponding to a uniform 8-element array of half-wave wires that are half-wavelength spaced, and the final result is that of Fig. 12 while Fig. 13 indicates the corresponding array configuration. Symmetry about the array center and about the plane containing the feed points is again assumed and the wire radii are all 0.007022λ. The result is quite good although convergence was considerably slower than in the case involving just the spacings. The results of Fig. 12 required about 11 minutes using an IBM 370/155 computer.

As a final illustration of the use of the analysis program described in Chapter 2 together with Rosenbrock's optimization procedure consider the design of a typical Yagi antenna. Here, lengths and spacings are to be determined for an array of parallel wires (with symmetry assumed only about the center-points of the array wires) such as to maximize the endfire gain $G_o$ defined as

$$G_o = \frac{4\pi \text{ (radiation intensity for the endfire direction)}}{\text{power input to the array}} \tag{10}$$

Formulas for computing the gain using matrices available from the analysis program are well-known [1,2]. These can be used with the iterative optimization procedure to optimize a Yagi antenna with respect to its wire lengths and spacings. The starting point can be an initial state corresponding to some classical or other improved design procedure. Figure 14 shows results after three iterations for Yagi antennas numbering 3-6 elements where

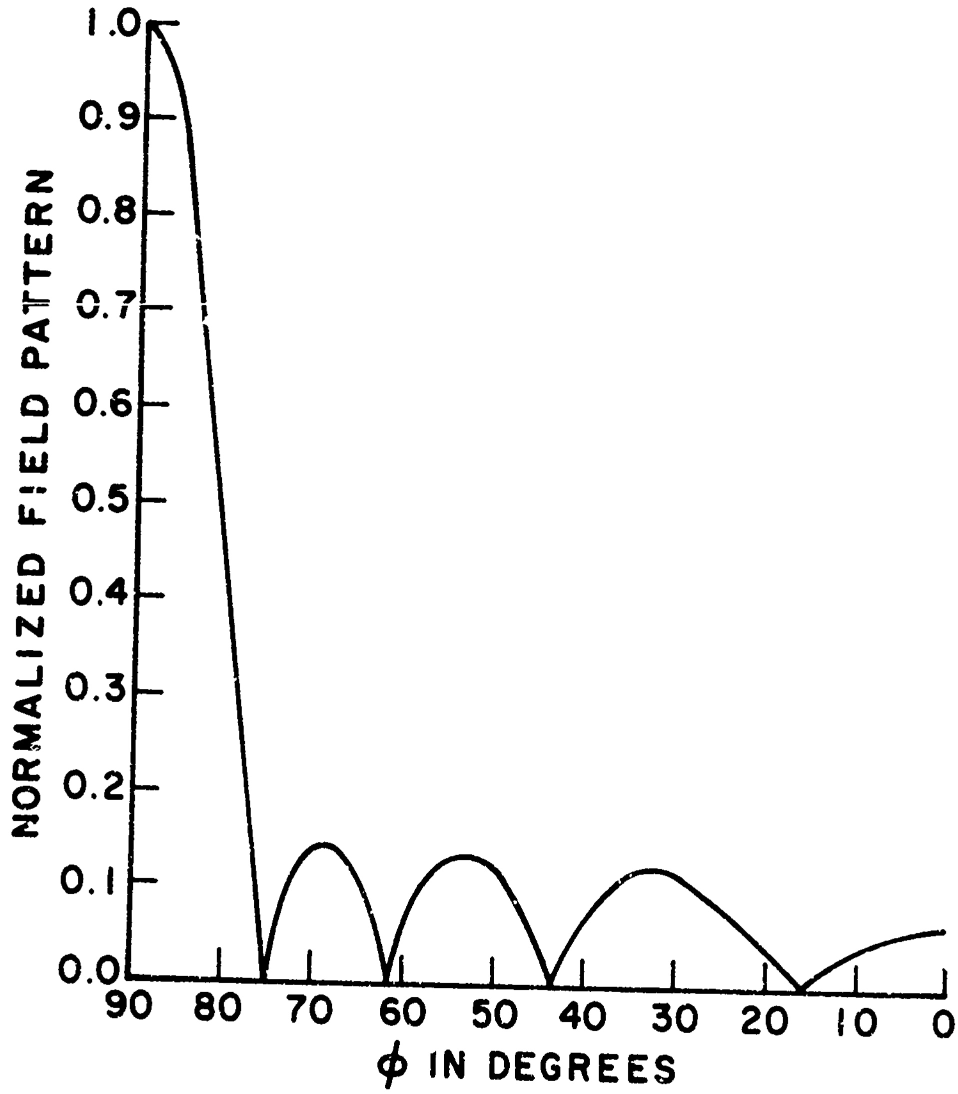



Fig. 12 - Pattern of an 8-element array with sidelobe level minimized with respect to both lengths and spacings.

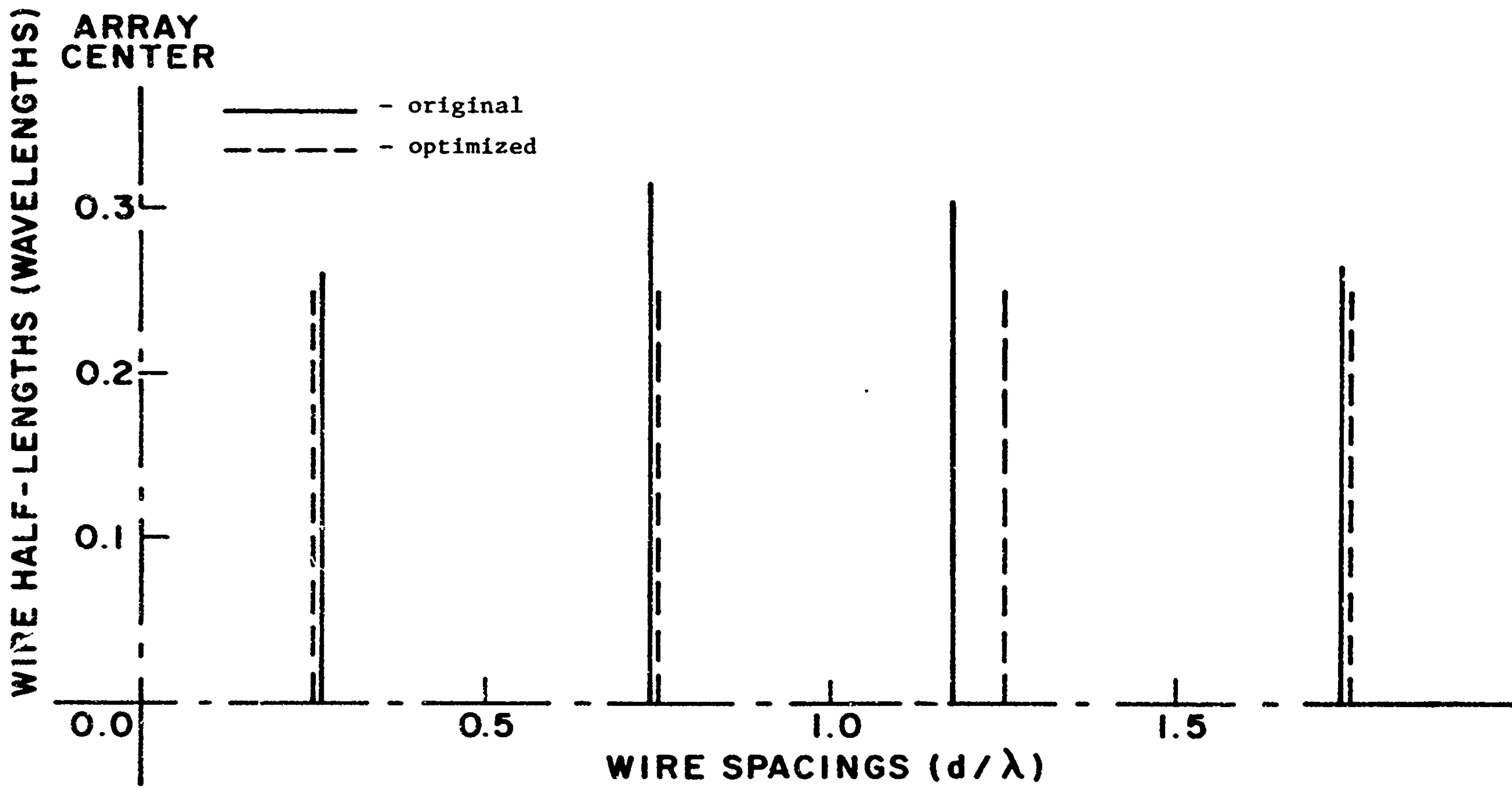



Fig. 13 - Array configuration corresponding to the pattern of Fig. 12.

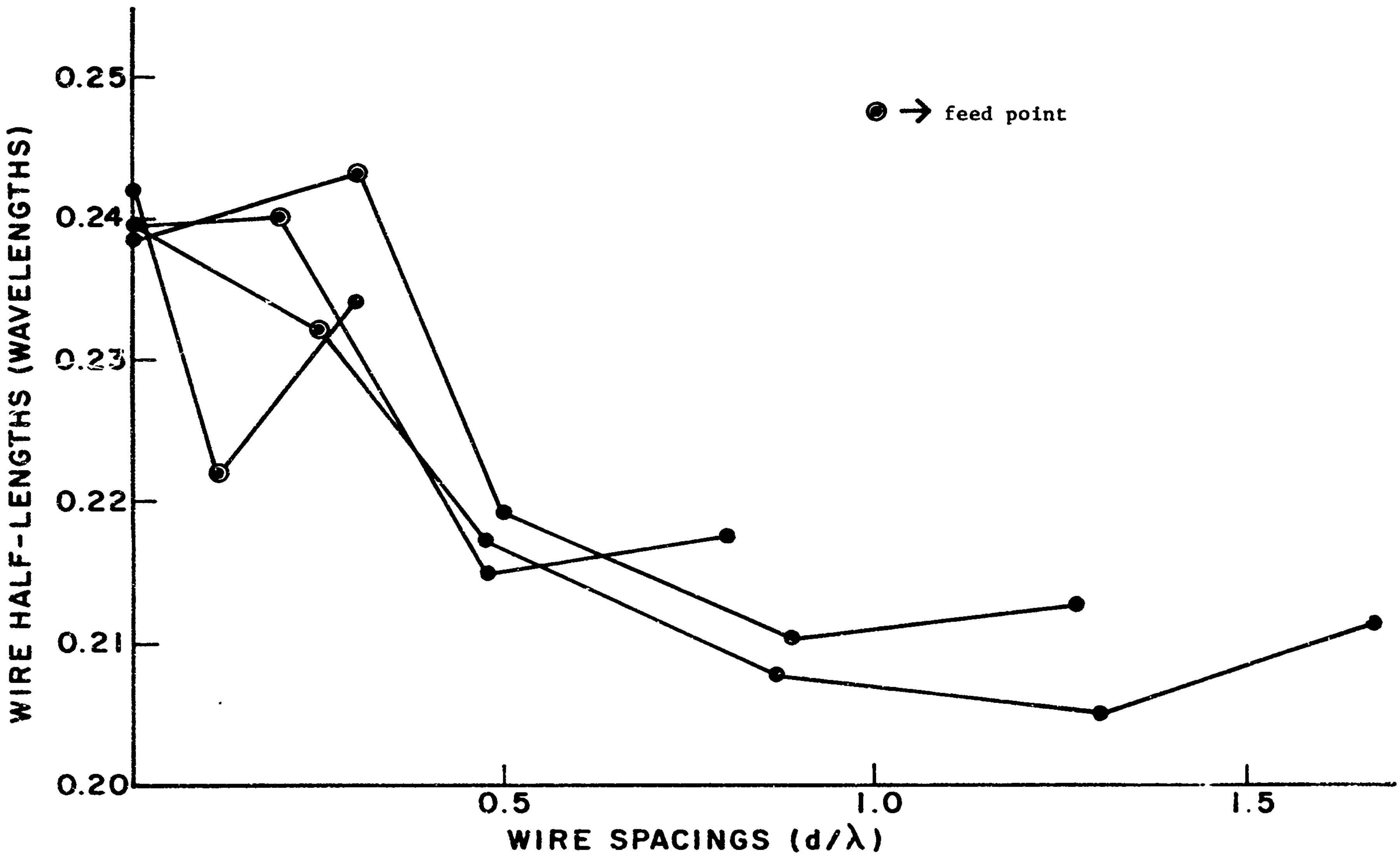



Fig. 14 – Gain maximization of Yagi arrays of 3-6 elements. Wire Radii are 0.01λ.

in each case the initial state (already the result of some gain-maximizing design procedure) was at least slightly improved upon. For the 3-element case the initial state was the result of a classical design procedure [15] and in the other cases the initial status were due to another analysis and optimization program based on matrix methods [16]. The time required to optimize the 6-wire Yagi antenna was 2 minutes on the IBM 370/155 computer. Improvements in gain were

a) from 5.9 to 9.1 for 3-element case, starting from an antenna designed by a standard technique [15]
b) from 14.2 to 14.5 for 4-element case, starting from an antenna optimized by matrix methods [16]
c) from 18.9 to 19.4 for 5-element case, starting from an antenna optimized by matrix methods [16]
d) from 24.4 to 24.9 for 6-element case, starting from an antenna optimized by matrix methods [16].

The examples presented in this chapter are by no means exhaustive and serve simply to illustrate possible uses of the analysis program described in Chapter 2 and presented in Appendix B. The examples do point out, however, that this particular analysis program can be used in solving quite difficult problems without requiring unreasonable computer time as is often the case.

# 4. A SPECIAL PROGRAM FOR HANDLING CERTAIN WIRE GROUND SYSTEMS

## 4.1 Introduction

This chapter describes a special user-oriented computer program that was designed specifically to handle certain problems involving vertical wire antennas over radial ground systems. The program, presented in Appendix C of this report, is capable of treating fairly general radiation problems for wire configurations such as depicted in Fig. 16. The program is a specialization of the general analysis program presented in an earlier Scientific

Report [4] that is capable of handling radiation problems involving arbitrary configurations of thin wires including junctions. By incorporating available symmetries the program presented here offers considerable advantage over the arbitrary wire program with regard to both speed of computation and simplicity.

The typical problem geometry is assumed to consist of a z-directed center wire plus a number NB of branches as in Fig. 15. The branches are

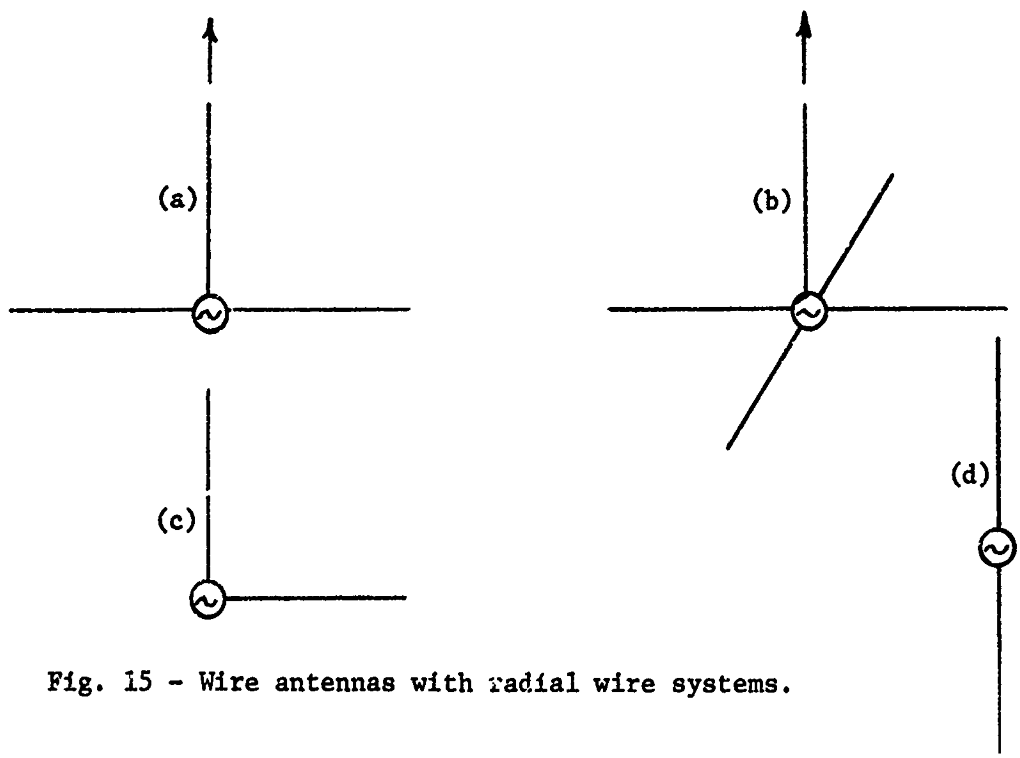



Fig. 15 - Wire antennas with radial wire systems.

assumed to be all the same length and all at the same angle with the center wire. The branches are uniformly distributed about the center wire, so that they are situated at regular intervals of $2\pi/NB$ radians.

As in the previous program all wires are assumed here to be perfect conductors. Each wire length L and radius a are such that $L/a >> 1$ and $a << \lambda$. Within Harrington's general procedure the so-called triangle functions are used for subsectional current expansion functions resulting in a piecewise linear approximation to the current of each wire. Triangle functions are also used for testing or weighting functions resulting in a Galerkin solution to the analysis problem which, as mentioned earlier, is

characterized by relatively fast convergence. The wires are assumed to be unloaded and the possible locations of applied excitations are restricted to points along the z-directed wire (see Fig. 15) which correspond to peaks of triangle current expansion functions. Most of the necessary details of the program were presented earlier with the arbitrary wire programs [4]. The only details included here are those that differ significantly from the previous work.

## 4.2 Data Input

All input data are provided through the main program. The first data statement reads in quantities NB, NEB, NEC, NR, BK, BLENTH, CLENTH, and ANGLE where

NB = the number of branches in the wire configuration not including the center wire. For example, in Fig. 15a NB = 2 while in Fig. 15b NB = 4.

NEB = the number of triangle expansion functions chosen for each branch. (All branches have the same length. The center wire is not included among the branches.)

NEC = the number of triangle expansion functions chosen for the center wire. (It should be noted with regard to both NEC and NEB that for accurate descriptions of current distributions it is advisable to use at least 12 triangle current expansion functions per wavelength of wire. On the other hand a smaller number may be used if only far-field patterns are of interest.)

BK = the factor $2\pi$ divided by the wavelength in meters.

BLENTH = the length in meters of each branch. (As mentioned earlier, all branches have the same length.)

CLENTH = the length in meters of the center wire.

ANGLE = the angle between the branches and the positive z-axis. (All branches are assumed to be at the same angle with the center wire.) This angle is expressed in degrees so that in Fig. 15a-c ANGLE = 90 and for Fig. 15d ANGLE = 180.

The second data statement provides the wire radius in meters. (For the example included with the program of Appendix C the wire radius is given as $10^{-4}$ meters.) The third reads in a number NF which equals the number of independent feed voltages to be applied to the center wire. As mentioned earlier these feed voltages must be applied at wire positions corresponding to peaks of triangle current expansion functions so that NF must not exceed the number of triangles assigned to the center wire. For radiation problems it is assumed NF $\geq$ 1, and DO LOOP 44 executes a total of NF times with an integer J1 and a complex number V read in using a single additional data card with each execution. J1 is the number of the particular triangle whose peak marks the desired position of the first excitation voltage and V is the desired value of that excitation in volts. Thus, following the data statement providing NF, the next NF data cards read in feed positions (corresponding to the numbers of specific triangles) and excitation voltages for each of the feed points along the center wire.

Far-field patterns are calculated and printed out using DO LOOP 17. Here, of course, it is necessary to provide the polar and azimuth angles of observation desired. In the example of the program included in Appendix C the instructions

```
DO 17 IPH = 1,2
PHI = (IPH-1)* 90
DO 17 ITH = 1,181,5
THE = ITH-1
```

cause field patterns to be computed in five degree steps of the polar angle $\theta$ for two different choices of azimuth angle $\phi = 0°, 90°$.

## 4.2 Program Description

Details of the problem geometry are calculated from the input data using subroutine GOMTRY and the generalized impedance matrix [Z] is computed using subroutine ZBRCH. [Z] is a square matrix of dimension equal to the number of independent expansion functions used, and its elements are functions only of the problem geometry. This matrix is inverted using sub-

routine LINEQ and the current matrix [I] is calculated by using this inverted matrix in (1) and DO LOOP 9. Finally, the field patterns specified are computed and printed out by way of DO LOOP 17 as mentioned earlier. In passing it should be pointed out that in the program presented in Appendix C the matrix $[Z]^{-1}$ occupies the same storage locations previously held by [Z]. Hence, the term [Z] denotes the generalized impedance matrix before inversion, and the generalized admittance matrix $[Z]^{-1}$ after inversion. It should also be noted that [Z] is manipulated in this program as a column matrix of $N^2$ complex numbers where N is the number of independent expansion functions used.

### 4.3 Examples

To illustrate use of this special purpose program consider the wire configurations shown in Fig. 15. For the antenna of Fig. 15a suppose the length of the center wire and the lengths of the branches are all $\lambda/4$.

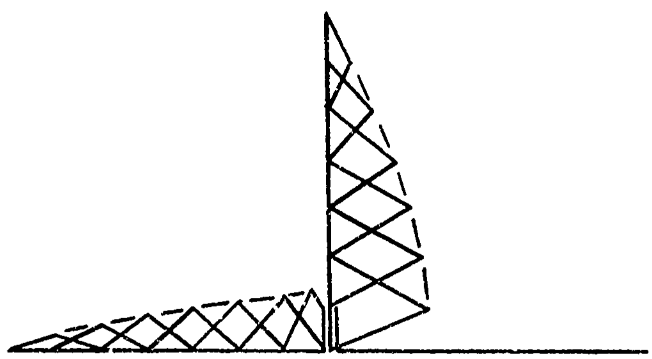


Fig. 16 - Junction treated as overlapping open-ended wires.

Suppose further that the wavelength is given to be one meter. Thus, BK = $2\pi$, CLENTH = 0.25 and BLENTH = 0.25. In accordance with previous discussions of the treatment of wire junctions [5] the antennas of Fig. 15 can be treated as systems of overlapping open-ended wires as shown in Fig. 16, corresponding to

the antenna of Fig. 15a. As indicated the antenna can be analyzed conveniently using six triangle current expansion functions for the center wire and seven for each branch, so that NEB = 7 and NEC = 6. Obviously, the branches are normal to the center wire in this case so that ANGLE = 90. The radius of each wire is $0.007\lambda$ so for the second data statement RAD = 0.007. Finally, the antenna is assumed to be base fed with a real unit excitation voltage. Hence, NF = 1 and since the base corresponds to the peak of the seventh triangle the fourth data statement provides J1 = 7 (locating the feed point) and an excitation given by V = 1.0 +j0.0 volts.

If the wire lengths and radii are the same as in Fig. 15a then the antennas of Figs. 15b-d can be handled with very simple changes in the input data. For example, if the antenna of Fig. 15b has a center wire and four branches, all of length $\lambda/4$, and if all remaining data are the same as before the only change required in the input is NB = 4. The vertical wire is always taken to be z-directed and the projection of the first branch on the x-y plane is always assumed to be x-directed, with the projections of the remaining branches distributed at equal-angle intervals. Thus, in Fig. 15b the first branch assumes the positive x-direction, the second the negative y-direction, and so on. Next, for the L-shaped antenna of Fig. 15c, it is obvious that the only change in input data required is NB = 1 if all other characteristics are the same as in the first example. Of course, in this case the only branch is +x-directed. Finally, the straight $\lambda/2$-wire of Fig. 15d can be treated as a $\lambda/4$ z-directed wire with a single branch of length $\lambda/4$ at an angle of 180° with the +z-axis. Here, input data differing from the specifications of Fig. 15a are NB = 1 and ANGLE = 180.

Current distributions for the antennas of Fig. 15 are plotted in Fig. 17. Input-output data, current distribution, and sample field patterns for an additional example are included with the program in Appendix C.

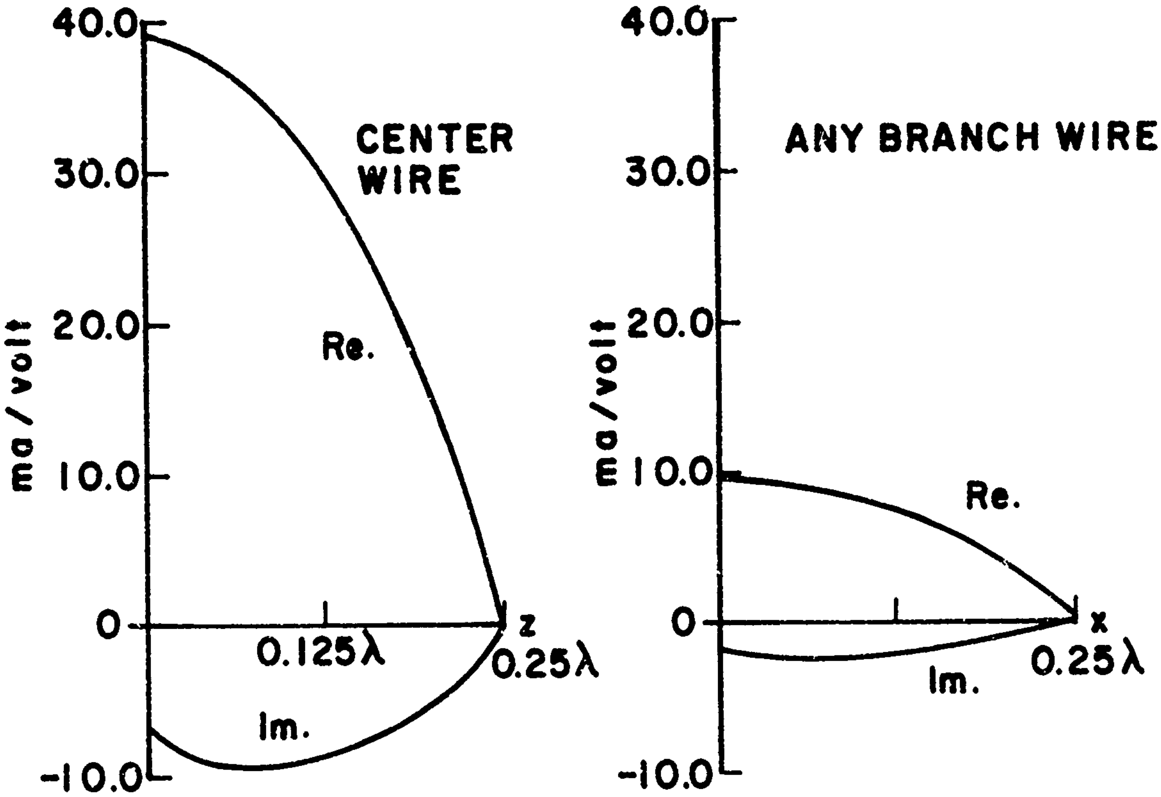



Fig. 17b - Current for Fig. 15b.

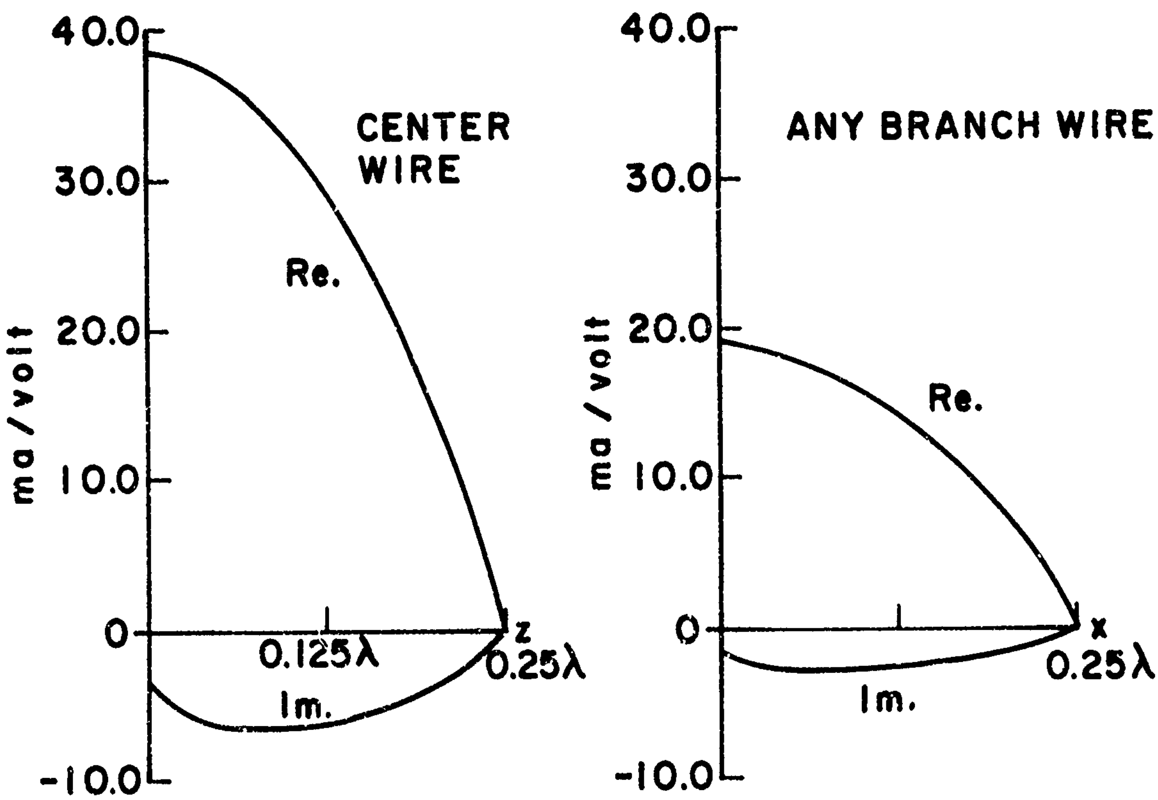



Fig. 17a - Current for Fig. 15a.

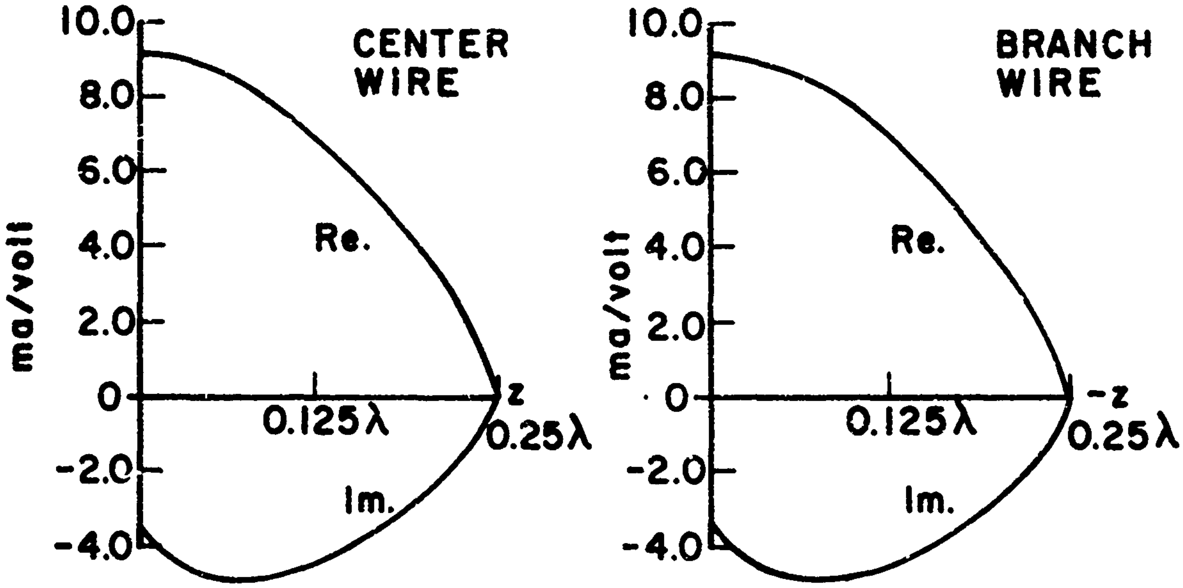



Fig. 17d - Current for Fig. 15d.

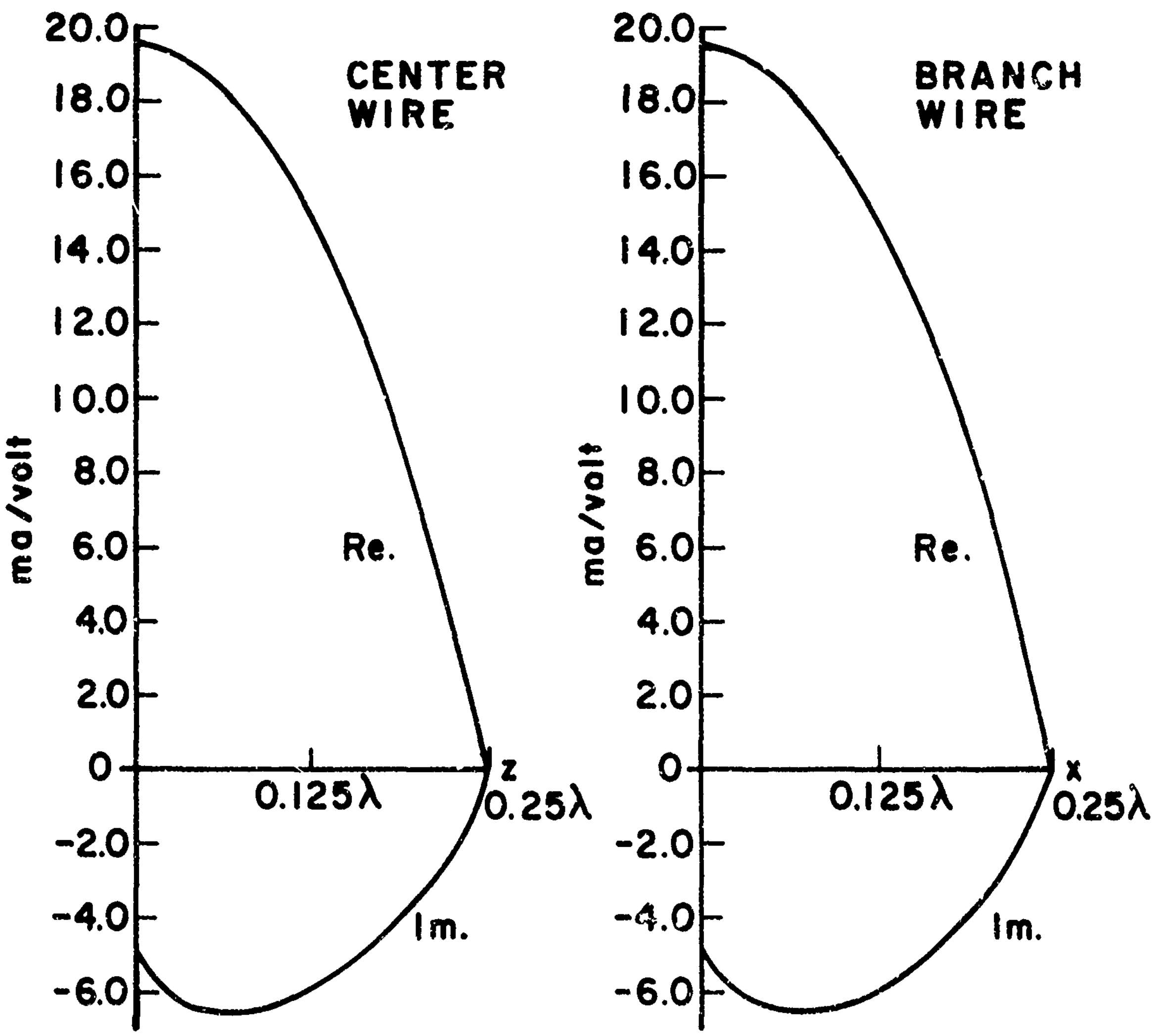



Fig. 17c - Current for Fig. 15c.

## 5. CONCLUSION

Two user-oriented computer programs have been presented and described for handling radiation problems involving certain special configurations of thin-wire antennas. Both of these stem from the method of moments with the first employing piecewise sinusoidal current expansion functions and the second a piecewise linear current approximation. The first treats radiation by arrays of thin, parallel, centerfed wires where the feed points all lie in the same plane. The wires are assumed to be lossless and all of the same radius. To illustrate use of the program several array design problems were treated. These include design of unequally spaced arrays to provide equal-sidelobe patterns, selection of wire lengths and spacings to optimize directivity, and reduction of pattern sidelobes through adjustments of wire lengths.

The second program presented here treats efficiently certain special situations involving vertical wire antennas over radial wire systems. Here again the wires are assumed lossless and all of the same radius, and feed points are restricted to the vertical wire. Several examples were included to illustrate use of this second program as well.

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## APPENDIX A

## EVALUATION OF THE ELEMENTS OF THE GENERALIZED IMPEDANCE MATRIX [Z]

For an array of straight, z-directed wires the elements of [Z] are given by (6) and can be evaluated as follows:

$$Z_{nm} = -\int_{n} \vec{I}_n \cdot \vec{E}_m \cdot dz$$

$$= -\left[\int_{z_{n-1}}^{z_n} \frac{\sin k(z-z_{n-1})}{\sin(k\Delta z_n)} + \int_{z_n}^{z_{n+1}} \frac{\sin k(z_{n+1}-z)}{\sin(k\Delta z_n)}\right] \times \frac{j30}{\sin(k\Delta z_m)}$$

$$\left[\frac{e^{-jkR_1}}{R_1} - 2\cos(k\Delta z_m)\frac{e^{-jkR_2}}{R_2} + \frac{e^{-jkR_3}}{R_3}\right] dz \qquad \text{(A1)}$$

where

$$R_1 = \sqrt{a^2 + (z-z_{m-1})^2}$$

$$R_2 = \sqrt{a^2 + (z-z_m)^2}$$

$$R_3 = \sqrt{a^2 + (z-z_{m+1})^2}$$

$$\Delta z_n = z_n - z_{n-1} = z_{n+1} - z_n$$

$$\Delta z_m = z_m - z_{m-1} = z_{m+1} - z_m$$

The computed elements are: $Z_{nm} = R_{nm} + jX_{nm}$ where,

$$R_{nm} = \frac{15}{\sin(k\Delta z_m)\ \sin(k\Delta z_n)} \times$$

$$[\cos k(z_{m-1} - z_{n-1})\ \{Ci(v_0) + Ci(u_0) - Ci(u_1) - Ci(v_1)\}$$

$$+ \sin k(z_{m-1} - z_{n-1})\ \{Si(v_0) - Si(u_0) + Si(u_1) - Si(v_1)\}$$

$$+ \cos k(z_{m+1} - z_{n-1})\ \{Ci(v_4) + Ci(u_4) - Ci(u_5) - Ci(v_5)\}$$

$$+ \sin k(z_{m+1} - z_{n-1})\ \{Si(v_4) - Si(u_4) + Si(u_5) - Si(v_5)\}$$

$$-2\ \cos(k\Delta z_m)\ \cos k(z_m - z_{n-1}\ \{Ci(v_2) + Ci(u_2) - Ci(u_3) - Ci(v_3)\}$$

$$-2\ \cos(k\Delta z_m) \cos k(z_m - z_{n-1})\ \ Si(v_2) - Si(u_2) + Si(u_3) - Si(v_3)\}$$

$$+ \cos k(z_{m-1} - z_{n+1})\ \{Ci(v_6) - Ci(v_1) + Ci(u_6) - Ci(u_1)\}$$

$$+ \sin k(z_{m-1} - z_{n+1})\ \{Si(v_6) - Si(u_6) + Si(u_1) - Si(v_1)\}$$

$$+ \cos k(z_{m+1} - z_{n+1})\ \{Ci(v_8) - Ci(v_5) - Ci(u_5) + Ci(u_8)\}$$

$$+ \sin k(z_{m+1} - z_{n+1})\ \{Si(v_8) - Si(u_8) + Si(u_5) - Si(v_5)\}$$

$$-2\ \cos(k\Delta z_m)\ \cos k(z_m - z_{n+1})\ \{Ci(v_7) - Ci(v_3) - Ci(u_3) + Ci(u_7)\}$$

$$-2\ \cos(k\Delta z_m)\ \sin k(z_m - z_{n+1})\ \{- Si(u_7) + Si(v_7)$$

$$+ Si(u_3) - Si(v_3)\}] \qquad \text{(A2)}$$

```
      A(3*(I-1)+J)=ZN+ZM
5     ZM=ZM+DZM
4     ZN=ZN+DZN
6     DO 3 I=1,9
      IF (A(I) .LT. 0.0) GO TO 21
      U(I)=SQRT(RR+A(I)*A(I))+A(I)
      V(I)=RR/U(I)
      GO TO 22
21    V(I)=SQRT(RR+A(I)*A(I))-A(I)
      U(I)=RR/V(I)
22    CALL    SICI(SI,CI,U(I))
      SU(I)=SI
      CU(I)=CI
      CALL    SICI(SI,CI,V(I))
      SV(I)=SI
      CV(I)=CI
      SP(I)=SU(I)+SV(I)
      SN(I)=SU(I)-SV(I)
      CP(I)=CU(I)+CV(I)
      CN(I)=CU(I)-CV(I)
3     CONTINUE
      C(1)=COS(A(1))
      C(2)=COS(A(2))
      C(3)=COS(A(3))

      C(7)=COS(A(7))
      C(8)=COS(A(8))
      C(9)=COS(A(9))
      S(1)=SIN(A(1))
      S(2)=SIN(A(2))
      S(3)=SIN(A(3))
      S(7)=SIN(A(7))
      S(8)=SIN(A(8))
      S(9)=SIN(A(9))
      RL=C(1)*(CP(1)-CP(4))-S(1)*(SN(4)-SN(1))+C(3)*(CP(3)-CP(6))-S(3)*
     $(SN(6)-SN(3))+C(7)*(CP(7)-CP(4))-S(7)*(SN(4)-SN(7))+C(9)*(CP(9)
     $-CP(6))-S(9)*(SN(6)-SN(9))-CC*(C(2)*(CP(2)-CP(5))-S(2)*(SN(5)
     $  -SN(2))+C(8)*(CP(8)-CP(5))-S(8)*(SN(5)-SN(8)))
      AG=C(1)*(SP(4)-SP(1))-S(1)*(CN(4)-CN(1))+C(3)*(SP(6)-SP(3))-S(3)*
     $(CN(6)-CN(3))+C(7)*(SP(4)-SP(7))-S(7)*(CN(4)-CN(7))+C(9)*(SP(6)-
     $SP(9))-S(9)*(CN(6)-CN(9))-CC*(C(2)*(SP(5)-SP(2))-S(2)*(CN(5)-CN(
     $2))+C(8)*(SP(5)-SP(8))-S(8)*(CN(5)-CN(8)))
      ZMN=CMPLX(RL,AG)*15./(SIN(DZN)*SIN(DZM))+ZMN
      IF(N.EQ.2 .OR. N.EQ.3) RETURN
      GO TO 10
      END
```

# APPENDIX B

## PROGRAM FOR LINEAR ARRAYS OF PARALLEL WIRES

This program is suitable for analyzing radiation by linear arrays of parallel thin-wire antennas. The wires can be of unequal lengths although all wire radii are assumed to be the same. Symmetry is assumed only about the plane containing the midpoints of the wires. The wires are assumed to be lossless with no externally applied loading. The sample output corresponds to analysis of an 8-element array of half-wave wires that are half-wavelength spaced and all of radius 0.01λ. Five expansion functions are used for approximating the current of each wire. (Because of symmetry only three of these are independent.) Each wire is fed with a real unit excitation and the principal H-plane pattern is computed in steps of three degrees.

The program is described in Chapter 2. The main program is presented first followed by the subroutines.

```
C
C     ***    MAIN  PROGRAM    ***
C
      DIMENSION    Y(20),VV(10)
      COMMON    AK,N,NL,N7,NV,VV,MO
      COMPLEX    CMPLX,CEXP,VV
      TP=6.2831853
C
C     AK  =  RADIUS  *  (2*PI)
C
      AK=0.01*TP
C
C     N = TOTAL NO. OF ELEMENTS IN THE LINEAR ARRAY
C
      N=8
C
C     NV = TOTAL NO. OF VARIABLES ( WIRE LENGTHS & INDEPENDENT ARRAY SPACINGS )
C
      NV=15
C
C     FIRST FEW ELEMENTS OF Y(I) DENOTE THE LENGTHS OF THE ELEMENTS AND
C
C     THE REST DENOTE THE DISTANCES OF THE ELEMENTS FROM THE FIRST, WHICH
C
C     IS CONSIDERED AS THE ORIGIN OF THE ARRAY.
```

```
C
C    *** NOTE ***  ALL DIMENSIONS OF Y(I) ARE METERS/WAVELENGTH * (2*PI)
C
C
      Y(1)=0.25*TP
      Y(2)=0.25*TP
      Y(3)=0.25*TP
      Y(4)=0.25*TP
      Y(5)=0.25*TP
      Y(6)=0.25*TP
      Y(7)=0.25*TP
      Y(8)=0.25*TP
      Y(9)=0.50*TP
      Y(10)=1.0*TP
      Y(11)=1.5*TP
      Y(12)=2.0*TP
      Y(13)=2.5*TP
      Y(14)=3.0*TP
      Y(15)=3.5*TP
C
C    VV(I)  SPECIFIES THE FEED VOLTAGES
C
      DO 3 I=1,N
      VV(I)=CMPLX(1.0,0.0)
    3 CONTINUE
C
C    PRINCIPAL H - PLANE PATTERN IS COMPUTED IN STEPS OF MQ DEGREES


       MQ=3
C
C          *****    END OF DATA    *****
C
       NZ=N+1
       NL=N-1
       CALL   ASCTED(Y)
       STOP
       END



       SUBROUTINE ASCTED(Y)
       COMPLEX    Z,V,SS,DETERM,CEXP,CMPLX,TERM,AZ,ZMN,YJ,ZM1,VV
       COMMON    AK,N,NL,NZ,NV,VV,MQ
       DIMENSION   M(15),YZ(10),DZ(15),Z(50,50),V(50,1),SS(50,1),CS(15)
       DIMENSION   TE(200),PH(200),Y(20),VV(10)
       BIG=0.0
       M(1)=0
       DO 2 I=1,N
C
C                 *****   NOTE   *****
C
C    FOR OPTIMIZATION PROBLEMS    MM=INT(Y(I)-0.571)+1
C
C    FOR ANALYSIS PROBLEMS    MM=INT(Y(I)-0.571)+3
```

```
C
C
      MM=INT(Y(I)-0.571)+3
      M(I+1)=M(I)+MM
 2    DZ(I)=Y(I)/FLOAT(MM)
      MN=M(N+1)
      DO 69 I=1,MN
 69   V(I,1)=CMPLX(0.0,0.0)
      DO 4 J=1,N
      M22=M(J)+1
      V(M22,1)=VV(J)
 4    CONTINUE
      YZ(1)=0.0
      DO 3 J=1,NL
 3    YZ(J+1)=Y(N+J)
      DO 86 L=1,N
      ZN=0.0
      ML1=M(L)+1
      ML2=M(L+1)
      DO 81 I=ML1,ML2
      ZM=0.0
      DO 82 J=ML1,ML2
      Z(I,J)=ZM1(ZM,ZN,DZ(L),AK)
 82   ZM=ZM+DZ(L)
 81   ZN=ZN+DZ(L)
 86   CONTINUE
      DO 74 K=1,NL
      K1=K+1
      ZN=-DZ(K)
      MK1=M(K)+1
      MK2=M(K+1)
      DO 71 I=MK1,MK2
      DO 73 L=K1,N
      ZM=-DZ(L)
      ML1=M(L)+1
      ML2=M(L+1)
      DO 72 J=ML1,ML2
      SD=YZ(L)-YZ(K)
      Z(I,J)=ZMN(ZM,ZN,DZ(L),DZ(K),SD)
 72   ZM=ZM+DZ(L)
 73   CONTINUE
 71   ZN=ZN+DZ(K)
 74   CONTINUE
      DO 64 K=2,N
      K1=K-1
      ZN=-DZ(K)
      MN1=M(K)+1
      MN2=M(K+1)
      DO 61 I=MN1,MN2
      DO 63 L=1,K1
      ZM=-DZ(L)
      ML1=M(L)+1
      ML2=M(L+1)
      DO 62 J=ML1,ML2
      SD=YZ(K)-YZ(L)
```

```
      Z(I,J)=ZMN(ZM,ZN,DZ(L),DZ(K),SD)
62    ZM=ZM+DZ(L)
63    CONTINUE
61    ZN=ZN+DZ(K)
64    CONTINUE
      CALL   CSMIN(Z,MN,DETERM)
      CALL   MULTPY(MN,MN,1,Z,V,SS)
      LL=0
      DO 11 LK=1,181,MQ
      PC=0.0174583*FLOAT(LK-1)
      PC=COS(PC)
      AZ=CMPLX(0.0,0.0)
      DO 44 J=1,N
      YJ=CMPLX(0.0,PC*YZ(J))
      TERM=CEXP(YJ)
      CS(J)=TAN(0.5*DZ(J))
      MC=M(J)+1
      MB=M(J+1)
      MA=MC+1
      AZ=AZ+SS(MC,1)*TERM*CS(J)
      IF(MA.GT.MB) GO TO 44
      DO 45 I=MA,MB
45    AZ=AZ+SS(I,1)*2.0*TERM*CS(J)
44    CONTINUE
      LL=LL+1
      TE(LL)=CABS(AZ)
      IF(TE(LL) .GT. BIG) BIG=TE(LL)
      PH(LL)=ATAN2(AIMAG(AZ),REAL(AZ))/0.0174533
11    CONTINUE
      DO 43 I=1,NV
      Y(I)=Y(I)/6.2831853
43    CONTINUE
      WRITE(3,111) (Y(I), I=1,N)
      WRITE(3,112) (Y(I), I=NZ,NV)
112   FORMAT('0',' SPACING= ',10F10.5///)
111   FORMAT('0',' LENGTH= ',10F10.5///)
      WRITE(3,892)
892   FORMAT('0',' CURRENT DISTRIBUTION ON THE ELEMENTS ')
      DO 49 K=1,N
      MK1=M(K)+1
      MK2=M(K+1)
      WRITE(3,895)
      WRITE(3,888)K
888   FORMAT('0',' CURRENT ON ',I5,5X,'ELEMENT'/)
      DO 50 NZ=MK1,MK2
      WRITE(3,999) SS(NZ,1)
999   FORMAT(' ',2F20.6)
50    CONTINUE
49    CONTINUE
      WRITE(3,895)
895   FORMAT('0'//)
      WRITE(3,893)
893   FORMAT('0','INPUT IMPEDANCES AT THE FEED POINTS'///)
      DO 51 J=1,N
      M22=M(J)+1
```

```
      IF(CABS(V(M22,1)). EQ. 0.0) GO TO 51
      AZ=V(M22,1)/SS(M22,1)
      WRITE(3,894) J,AZ
 894  FORMAT('0','INPUT IMPEDANCE OF',I5,5X,'ELEMENT =',2F20.6//)
 51   CONTINUE
      WRITE(3,891)
 891  FORMAT('1',' NORMALIZED E- FIELD PATTERN '///)
      LM=0
      DO 95 L=1,LL
      TE(L)=TE(L)/BIG
      WRITE(3,342)LM,TE(L),PH(L)
 342  FORMAT(' ',I5,2F40.6)
      LM=LM+MO
 95   CONTINUE
      WRITE(3,890) BIG
 890  FORMAT('0',' NORMALIZATION CONSTANT =',E15.5///)
      RETURN
      END



      FUNCTION   ZMN(ZM1,ZN1,DZM,DZN,R)
C
C   COMPUTES MUTUAL IMPEDANCE BETWEEN TWO ELEMENTS OF LENGTHS DZM AND DZN
C
C   EACH OF RADIUS  R  HAVING STARTING POINTS AT ZM1 AND ZN1
C
C   *** NOTE ***   ALL DIMENSIONS ARE  (*2*PI)
C
C
      COMPLEX   ZMN,CMPLX
      DIMENSION A(9),U(9),V(9),SU(9),SV(9),CU(9),CV(9),C(9),S(9),SP(9),
     $SN(9),CN(9),CP(9)
      N=1
      IF(ZM1.EQ.-DZM) N=2
      CC=2.0*COS(DZM)
      RR=R*R
      ZN=ZN1
      DO 1 I=1,3
      ZM=ZM1
      DO 2 J=1,3
      A(3*(I-1)+J)=ZN-ZM
 2    ZM=ZM+DZM
 1    ZN=ZN+DZN
      ZMN=CMPLX(0.0,0.0)
      GO TO 6
 10   ZN=ZN1
      N=3
      DO 4 I=1,3
      ZM=ZM1
      DO 5 J=1,3
```

```
      A(3*(I-1)+J)=ZN+ZM
5     ZM=ZM+DZM
4     ZN=ZN+DZN
6     DO 3 I=1,9
      IF (A(I) .LT. 0.0) GO TO 21
      U(I)=SQRT(RR+A(I)*A(I))+A(I)
      V(I)=RR/U(I)
      GO TO 22
21    V(I)=SQRT(RR+A(I)*A(I))-A(I)
      U(I)=RR/V(I)
22    CALL   SICI(SI,CI,U(I))
      SU(I)=SI
      CU(I)=CI
      CALL   SICI(SI,CI,V(I))
      SV(I)=SI
      CV(I)=CI
      SP(I)=SU(I)+SV(I)
      SN(I)=SU(I)-SV(I)
      CP(I)=CU(I)+CV(I)
      CN(I)=CU(I)-CV(I)
3     CONTINUE
      C(1)=COS(A(1))
      C(2)=COS(A(2))
      C(3)=COS(A(3))

      C(7)=COS(A(7))
      C(8)=COS(A(8))
      C(9)=COS(A(9))
      S(1)=SIN(A(1))
      S(2)=SIN(A(2))
      S(3)=SIN(A(3))
      S(7)=SIN(A(7))
      S(8)=SIN(A(8))
      S(9)=SIN(A(9))
      RL=C(1)*(CP(1)-CP(4))-S(1)*(SN(4)-SN(1))+C(3)*(CP(3)-CP(6))-S(3)*
     $(SN(6)-SN(3))+C(7)*(CP(7)-CP(4))-S(7)*(SN(4)-SN(7))+C(9)*(CP(9)
     $-CP(6))-S(9)*(SN(6)-SN(9))-CC*(C(2)*(CP(2)-CP(5))-S(2)*(SN(5)
     $ -SN(2))+C(8)*(CP(8)-CP(5))-S(8)*(SN(5)-SN(8)))
      AG=C(1)*(SP(4)-SP(1))-S(1)*(CN(4)-CN(1))+C(3)*(SP(6)-SP(3))-S(3)*
     $(CN(6)-CN(3))+C(7)*(SP(4)-SP(7))-S(7)*(CN(4)-CN(7))+C(9)*(SP(6)-
     $SP(9))-S(9)*(CN(6)-CN(9))-CC*(C(2)*(SP(5)-SP(2))-S(2)*(CN(5)-CN(
     $2))+C(8)*(SP(5)-SP(8))-S(8)*(CN(5)-CN(8)))
      ZMN=CMPLX(RL,AG)*15./(SIN(DZN)*SIN(DZM))+ZMN
      IF(N.EQ.2 .OR. N.EQ.3) RETURN
      GO TO 10
      END
```

```
      FUNCTION      ZM1(ZM,ZN,DK,AK)
C
C     COMPUTES MUTUAL IMPEDANCE BETWEEN TWO ELEMENTS OF RADIUS AK AND
C
C     LENGTH DK HAVING STARTING POINTS AT ZM AND ZN
C
C     *** NOTE ***    ALL DIMENSIONS ARE   (*2*PI)
C
C
      COMPLEX    CMPLX,ZM1
      CDK=COS(DK)
      SDK=15./(1.-CDK*CDK)
      DSQ=AK*AK
      D4=4.*CDK
      DD4=2.+D4*CDK
      N=1
      IF(ZM.EQ.C.C) N=C
      A=ZN-ZM
      B=A-DK
      C=A+DK
      D=B-DK
      E=C+DK
      CA=COS(A)
      SA=SIN(A)
      CB=COS(B)
      SB=SIN(B)
      CC=COS(C)
      SC=SIN(C)
      CD=COS(D)
      SD=SIN(D)
      CE=COS(E)
      SE=SIN(E)
      U0=SQRT(DSQ+A*A)+A
      V0=DSQ/U0
      U1=SQRT(DSQ+C*C)+C
      V1=DSQ/U1
      U2=SQRT(DSQ+B*B)+B
      V2=DSQ/U2
      U4=SQRT(DSQ+D*D)+D
      V4=DSQ/U4
      U6=SQRT(DSQ+E*E)+E
      V6=DSQ/U6
      CALL    SICI(SI,CI,U0)
      SU0=SI
      CU0=CI
      CALL    SICI(SI,CI,V0)
      SV0=SI
      CV0=CI
      CALL    SICI(SI,CI,U1)
      SU1=SI
      CU1=CI
      CALL    SICI(SI,CI,V1)
      SV1=SI
      CV1=CI
```

```
      CALL    SICI(SI,CI,U2)
      SU2=SI
      CU2=CI
      CALL    SICI(SI,CI,V2)
      SV2=SI
      CV2=CI
      CALL    SICI(SI,CI,U4)
      SU4=SI
      CU4=CI
      CALL    SICI(SI,CI,V4)
      SV4=SI
      CV4=CI
      CALL    SICI(SI,CI,U6)
      SU6=SI
      CU6=CI
      CALL    SICI(SI,CI,V6)
      SV6=SI
      CV6=CI
      RL=DD4*(CA*(CUC+CVO)+(SUO-SVO)*SA)-D4*(CC*(CU1+CV1)+CB*(CU2+CV2)+
     $SC*(SU1-SV1)+SB*(SU2-SV2))+CD*(CU4+CV4)+SD*(SU4-SV4)+CE*(CU6+CV6)
     $+SF*(SU6-SV6)
      AG=SE*(CU6-CV6)-CF*(SU6+SV6)+SD*(CU4-CV4)-CD*(SU4+SV4)-D4*(SC*
     $(CU1-CV1)+SB*(CU2-CV2)-CC*(SU1+SV1)-CB*(SU2+SV2))+DD4*(SA*(CUO-
     $CVO)-CA*(SUO+SVO))
      ZM1=CMPLX(RL,AG)*SDK
      IF(N.EQ.0) RETURN
      A=ZN+ZM
      B=A-DK
      C=A+DK
      D=B-DK
      E=C+DK
      CA=COS(A)
      SA=SIN(A)
      CB=COS(B)
      SB=SIN(B)
      CC=COS(C)
      SC=SIN(C)
      CD=COS(D)
      SD=SIN(D)
      CE=COS(E)
      SE=SIN(F)
      VO=SQRT(DSQ+D*D)+D
      UO=DSQ/VO
      V1=SQRT(DSQ+B*B)+B
      U1=DSQ/V1
      V2=SQRT(DSQ+A*A)+A
      U2=DSQ/V2
      V4=SQRT(DSQ+C*C)+C
      U4=DSQ/V4
      V6=SQRT(DSQ+E*E)+F
      U6=DSQ/V6
      CALL    SICI(SI,CI,UO)
      SUO=SI
      CUO=CI
```

```
      CALL    SICI(SI,CI,V0)
      SV0=SI
      CV0=CI
      CALL    SICI(SI,CI,U1)
      SU1=SI
      CU1=CI
      CALL    SICI(SI,CI,V1)
      SV1=SI
      CV1=CI
      CALL    SICI(SI,CI,U2)
      SU2=SI
      CU2=CI
      CALL    SICI(SI,CI,V2)
      SV2=SI
      CV2=CI
      CALL    SICI(SI,CI,U4)
      SU4=SI
      CU4=CI
      CALL    SICI(SI,CI,V4)
      SV4=SI
      CV4=CI
      CALL    SICI(SI,CI,U6)
      SU6=SI
      CU6=CI
      CALL    SICI(SI,CI,V6)
      SV6=SI
      CV6=CI
      RL=DD4*(CA*(CU2+CV2)-SA*(SU2-SV2))-D4*(CC*(CU4+CV4)+CB*(CU1+CV1)-
     $SC*(SU4-SV4)-SB*(SU1-SV1))+CD*(CU0+CV0)-SD*(SU0-SV0)+CE*(CU6+CV6)
     $-SF*(SU6-SV6)
      AG=-SF*(CU6-CV6)-CE*(SU6+SV6)-SD*(CU0-CV0)-CD*(SU0+SV0)+D4*(SC*(
     $CU4-CV4)+SB*(CU1-CV1)+CB*(SU1+SV1)+CC*(SU4+SV4))+DD4*(-CA*(SU2+
     $SV2)-SA*(CU2-CV2))
      ZM1=CMPLX(RL,AG)*SDK+ZM1
      RETURN
      END


      SUBROUTINE SICI(SI,CI,X)
C
C     COMPUTES SINE AND COSINE INTEGRALS
C
      Z=ABS(X)
      IF(Z-4.)1,1,4
    1 Y=(4.-Z)*(4.+Z)
    3 SI=X*(((((1.753141E-9*Y+1.568988E-7)*Y+1.374168E-5)*Y+6.939889E-4)
     1*Y+1.964882E-2)*Y+4.395509E-1)
      CI=((5.772156E-1+ALOG(Z))/Z-Z*(((((1.386985E-10*Y+1.584996E-8)*Y
     1+1.725752E-6)*Y+1.185999E-4)*Y+4.990920E-3)*Y+1.315308E-1))*Z
      RETURN
    4 SI=SIN(Z)
      Y=COS(Z)
      Z=4./Z
      U=((((((((4.048069E-3*Z-2.279143E-2)*Z+5.515070E-2)*Z-7.261642E-2)
```

```
     1*Z+4.987716E-2)*Z-3.332519E-3)*Z-2.314617E-2)*Z-1.134958E-5)*Z
     2+6.250011E-2)*Z+2.583989E-10
      V=(((((((((-5.108699E-3*Z+2.819179E-2)*Z-6.537283E-2)*Z
     1+7.902034E-2)*Z-4.400416E-2)*Z-7.945556E-3)*Z+2.601293E-2)*Z
     2-3.764000E-4)*Z-3.122418E-2)*Z-6.646441E-7)*Z+2.500000E-1
      CI=Z*(SI*V-Y*U)
      SI=-Z*(SI*U+Y*V)        +1.570796
      RETURN
      END


      SUBROUTINE   CSMIN(A,N,DETERM)
C
C     INVERTS A GIVEN SQUARE MATRIX
C
      COMPLEX   A,PIVOT,AMAX,T,SWAP,DETERM,U,CMPLX,CONJ
      DIMENSION   IPIVOT(50),INDEX(50,2),A(50,50),PIVOT(50)
      DETERM=CMPLX(1.0,0.0)
      DO 20 J=1,N
 20   IPIVOT(J)=0
      DO 600 I=1,N
      AMAX=CMPLX(0.0,0.0)
      DO 105 J=1,N
      IF(IPIVOT(J)-1) 60,105,60
 60     DO 100 K=1,N
      IF(IPIVOT(K)-1) 80,100,740
 80   TEMP=AMAX*CONJ (AMAX)-A(J,K)*CONJ (A(J,K))
      IF(TEMP)85,85,100
 85   IROW=J
      ICOLUM=K
      AMAX=A(J,K)
 100  CONTINUE
 105  CONTINUE
      IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
      IF(IROW-ICOLUM) 140,260,140
 140  DETERM=-DETERM
      DO 200 L=1,N
      SWAP=A(IROW,L)
      A(IROW,L)=A(ICOLUM,L)
 200  A(ICOLUM,L)=SWAP
 260  INDEX(I,1)=IROW
      INDEX(I,2)=ICOLUM
      PIVOT(I)=A(ICOLUM,ICOLUM)
      DETERM=DETERM*PIVOT(I)
      TEMP=PIVOT(I)*CONJ (PIVOT(I))
      IF(TEMP) 330,720,330
 330  A(ICOLUM,ICOLUM)=CMPLX(1.0,0.0)
      DO 350 L=1,N
      U=PIVOT(I)
 350  A(ICOLUM,L)=A(ICOLUM,L)/U
 380  DO 550 L1=1,N
      IF(L1-ICOLUM)400,550,400
```

```
400   T=A(L1,ICOLUM)
      A(L1,ICOLUM)=CMPLX(0.0,0.0)
      DO 450 L=1,N
      U=A(ICOLUM,L)
450   A(L1,L)=A(L1,L)-U*T
550   CONTINUE
600   CONTINUE
      DO 710 I=1,N
      L=N+1-I
      IF(INDEX(L,1)-INDEX(L,2)) 630,710,630
630   JROW=INDEX(L,1)
      JCOLUM=INDEX(L,2)
      DO 705 K=1,N
      SWAP=A(K,JROW)
      A(K,JROW)=A(K,JCOLUM)
      A(K,JCOLUM)=SWAP
705   CONTINUE
710   CONTINUE
      RETURN
720   WRITE(3,730)
730   FORMAT('0',5X,' MATRIX  IS  SINGULAR '//)
740   RETURN
      END
```

```
      FUNCTION   CONJ(ZZ)
      COMPLEX   CMPLX,ZZ,CONJ
      CONJ=CMPLX(REAL(ZZ),-AIMAG(ZZ))
      RETURN
      END
```

```
      SUBROUTINE   MULTPY(L,M,N,A,B,C)
C
C   MULTIPLIES TWO MATRICES
C
      COMPLEX   A(50,50),B(50,1),C(50,1)
      DO 1 I=1,L
      DO 1K=1,N
      C(I,K)=0.0
      DO 1 J=1,M
1     C(I,K)=C(I,K)+A(I,J)*B(J,K)
      RETURN
      END
```

LENGTH= 0.25000 0.25000 0.25000 0.25000 0.25000 0.25000 0.25000 0.25000

SPACING= 0.50000 1.00000 1.50000 2.00000 2.50000 3.00000 3.50000

CURRENT DISTRIBUTION ON THE ELEMENTS

CURRENT ON 1 ELEMENT

| | |
|---|---|
| 0.135590E-01 | -0.727654E-03 |
| 0.120027E-01 | -0.296037E-02 |
| 0.792061E-02 | -0.246207E-02 |

CURRENT ON 2 ELEMENT

| | |
|---|---|
| 0.174993E-01 | 0.107518E-02 |
| 0.154946E-01 | -0.136203E-02 |
| 0.102342E-01 | -0.140119E-02 |

CURRENT ON 3 ELEMENT

| | |
|---|---|
| 0.165198E-01 | 0.110923E-02 |
| 0.146296E-01 | -0.133398E-02 |
| 0.966777E-02 | -0.138786E-02 |

CURRENT ON 4 ELEMENT

| | |
|---|---|
| 0.168413E-01 | 0.107989E-02 |
| 0.149134E-01 | -0.135921E-02 |
| 0.985340E-02 | -0.140274E-02 |

CURRENT ON 5 ELEMENT

| | |
|---|---|
| 0.168413E-01 | 0.108001E-02 |
| 0.149135E-01 | -0.135910E-02 |
| 0.985348E-02 | -0.140265E-02 |

CURRENT ON 6 ELEMENT

| | |
|---|---|
| 0.165198E-01 | 0.110928E-02 |
| 0.146296E-01 | -0.133391E-02 |
| 0.966777E-02 | -0.138782E-02 |

| CURRENT ON 7 ELEMENT | | CURRENT ON 8 ELEMENT | |
|---|---|---|---|
| 0.174993E-01 | 0.107520E-02 | 0.135590E-01 | -0.727606E-03 |
| 0.154946E-01 | -0.136201E-02 | 0.120028E-01 | -0.296031E-02 |
| 0.102342E-01 | -0.140117E-02 | 0.792063E-02 | -0.246204E-02 |

INPUT IMPEDANCES AT THE FEED POINTS

| | | | |
|---|---|---|---|
| INPUT IMPEDANCE OF 1 ELEMENT = | 0.735400E 02 | 0.394658E 01 |
| INPUT IMPEDANCE OF 2 ELEMENT = | 0.569303E 02 | -0.349788E 01 |
| INPUT IMPEDANCE OF 3 ELEMENT = | 0.602618E 02 | -0.404629E 01 |
| INPUT IMPEDANCE OF 4 ELEMENT = | 0.591347E 02 | -0.379179E 01 |
| INPUT IMPEDANCE OF 5 ELEMENT = | 0.591345E 02 | -0.379221E 01 |
| INPUT IMPEDANCE OF 6 ELEMENT = | 0.602617E 02 | -0.404646E 01 |
| INPUT IMPEDANCE OF 7 ELEMENT = | 0.569303E 02 | -0.349793E 01 |
| INPUT IMPEDANCE OF 8 ELEMENT = | 0.735398E 02 | 0.394630E 01 |

NORMALIZED F- FIELD PATTERN

| | | |
|---|---|---|
| 0 | 0.000001 | 53.712891 |
| 3 | 0.001453 | 66.059052 |
| 6 | 0.005793 | 63.462265 |
| 9 | 0.012975 | 59.139740 |
| 12 | 0.022816 | 53.071182 |
| 15 | 0.034886 | 45.249466 |
| 18 | 0.048338 | 35.656738 |
| 21 | 0.061772 | 24.258026 |
| 24 | 0.073137 | 10.986127 |
| 27 | 0.079783 | -4.302690 |
| 30 | 0.078781 | -21.962402 |
| 33 | 0.067565 | -43.012985 |
| 36 | 0.045304 | -71.680725 |
| 39 | 0.020482 | -141.776978 |
| 42 | 0.042617 | 118.120651 |
| 45 | 0.083445 | 80.080215 |
| 48 | 0.115672 | 50.298401 |
| 51 | 0.126299 | 21.032181 |
| 54 | 0.105935 | -10.648551 |
| 57 | 0.053941 | -54.991577 |
| 60 | 0.044821 | 161.263107 |
| 63 | 0.129797 | 106.062408 |
| 66 | 0.198734 | 70.245300 |
| 69 | 0.214862 | 36.055420 |
| 72 | 0.151932 | -0.682838 |
| 75 | 0.030137 | -121.462845 |
| 78 | 0.236566 | 131.869217 |
| 81 | 0.503893 | 95.018326 |
| 84 | 0.755990 | 60.819748 |
| 87 | 0.935622 | 27.297760 |
| 90 | 1.000000 | -5.863531 |
| 93 | 0.933421 | -38.657883 |
| 96 | 0.752121 | -70.907059 |
| 99 | 0.499261 | -102.105713 |
| 102 | 0.232193 | -130.024353 |
| 105 | 0.029792 | -81.258011 |
| 108 | 0.153752 | -29.985626 |
| 111 | 0.215196 | -55.546463 |
| 114 | 0.197901 | -82.309097 |
| 117 | 0.128347 | -105.952866 |
| 120 | 0.043571 | -107.648117 |
| 123 | 0.055012 | -20.748306 |
| 126 | 0.106574 | -31.308990 |
| 129 | 0.126362 | -52.028732 |
| 132 | 0.115269 | -72.925568 |
| 135 | 0.082777 | -90.934418 |
| 138 | 0.041938 | -97.939575 |
| 141 | 0.020670 | -39.324295 |
| 144 | 0.045762 | -10.939811 |
| 147 | 0.067850 | -19.835052 |

| | | |
|---|---|---|
| 150 | 0.078877 | -33.298767 |
| 153 | 0.079726 | -47.126251 |
| 156 | 0.072973 | -60.234039 |
| 159 | 0.061552 | -72.199875 |
| 162 | 0.048101 | -82.808685 |
| 165 | 0.034663 | -91.933441 |
| 168 | 0.022626 | -99.493362 |
| 171 | 0.012829 | -105.420761 |
| 174 | 0.005694 | -109.682693 |
| 177 | 0.001400 | -112.246414 |
| 180 | 0.000301 | -96.129089 |

NORMALIZATION CONSTANT = 0.13669E 00

# APPENDIX C

## PROGRAM FOR WIRE OVER RADIAL WIRE SYSTEM

This program is suitable for analyzing radiation by wires over radial wire systems as exemplified by Fig. 15. The wires are assumed lossless with no externally applied loading and excitations are restricted to center-wire positions corresponding to peaks of triangle current expansion functions. The first triangle begins at the junction and extends towards the end of the center wire. The junction itself corresponds to the peak of the first triangle of the branch wire. The sample output is for a $\lambda/4$ center wire with two branches, each of length $\lambda/8$. The branches are normal to the center wire as in Fig. 15a. The center wire has six triangle expansion functions while each branch supports four. The wavelength is one meter and each wire is of radius $10^{-4}$ meter. The configuraitcn has a single unit excitation and that is located right at the junction itself. Since the junction marks the position of the first branch triangle which is the seventh triangle the feed point is assigned the number seven. Patterns are computed for the $\phi = 0°$, $90°$ planes. x,y,z-coordinates of axial points defining the problem geometry are labeled PX, PY, PZ, respectively in the sample output.

Subroutines are listed first, followed by the main program and typical output. The program is described in Chapter 4. Further information on computational procedures is available elsewhere [4].

```
      SUBROUTINE LINEQ (N,Z)
      COMPLEX Z( 400),STOR,STO,ST,S
      DIMENSION LA(50)
      DO 20 I=1,N
      LA(I)=I
   20 CONTINUE
      M1=0
      DO 18 M=1,N
      K=M
      DO 2 I=M,N
      K1=M1+I
      K2=M1+K
      IF (CABS(Z(K1))-CABS(Z(K2))) 2,2,6
    6 K=I
    2 CONTINUE
      LS=LA(M)
      LA(M)=LA(K)
      LA(K)=LS
```

```
      K2=M1+K
      STOR=Z(K2)
      J1=0
      DO 7 J=1,N
      K1=J1+K
      K2=J1+M
      STO=Z(K1)
      Z(K1)=Z(K2)
      Z(K2)=STO/STOR
      J1=J1+N
    7 CONTINUE
      K1=M1+M
      Z(K1)=1./STOR
      DO 11 I=1,N
      IF(I-M) 12,11,12
   12 K1=M1+I
      ST=Z(K1)
      Z(K1)=0.
      J1=0
      DO 10 J=1,N
      K2=J1+M
      K1=J1+I
      Z(K1)=Z(K1)-Z(K2)*ST
      J1=J1+N
   10 CONTINUE
   11 CONTINUE
      M1=M1+N
   18 CONTINUE
      J1=0
      DO 9 J=1,N
      IF (J-LA(J)) 14,8,14
   14 LAJ=LA(J)
      J2=(LAJ-1)*N
   31 DO 13 I=1,N
      K2=J2+I
      K1=J1+I
      S=Z(K2)
      Z(K2)=Z(K1)
      Z(K1)=S
   13 CONTINUE
      LA(J)=LA(LAJ)
      LA(LAJ) =LAJ
      IF (J-LA(J)) 14,8,14
    8 J1=J1+N
    9 CONTINUE
      RETURN
      END
C
      SUBROUTINE ROW(N,TH,PH,E)
      COMPLEX C(20),E(2),U,U1,U2,U3,U4,U5
      COMMON XX(50),XY(50),XZ(50),TX(50),TY(50),TZ(50),AL(50)
      COMMON T(100),TP(100),RAD2(10),L(10),LL(10),LR(10),BK
      COMMON /COA/ C
      DIMENSION BKR( 50),DT( 50),DP( 50)
      U=(0.,1.)
      ETA=376.7307
```

```
      CT=COS(TH)
      ST=SIN(TH)
      CP=COS(PH)
      SP=SIN(PH)
      S1=CT*CP
      S2=CT*SP
      BK1=BK*ST*CP
      BK2=BK*ST*SP
      BK3=BK*CT
      N2=1
      N3=-2
      DO 1 NS=1,N
      IF (L(N2)-NS) 2,3,2
    3 KK=1
      N3=N3+2
      N2=N2+1
      GO TO 4
    2 KK=3
    4 DO 5 K=KK,4
      N7=N3+K
      BKR(N7)=XX(N7)*BK1+XY(N7)*BK2+XZ(N7)*BK3
      DT(N7)=TX(N7)*S1+TY(N7)*S2-TZ(N7)*ST
      DP(N7)=-TX(N7)*SP+TY(N7)*CP
    5 CONTINUE
      N3=N3+2
    1 CONTINUE
      N2=1
      N3=-2
      U3=0.
      U4=0.
      DO 6 NS=1,N
      IF (L(N2)-NS) 7,8,7
    8 N3=N3+2
      N2=N2+1
    7 J1=(NS-1)*4
      U1=0.
      U2=0.
      DO 9 JS=1,4
      J2=J1+JS
      J3=N3+JS
      S1=BKR(J3)
      U5=T(J2)*(COS(S1)+U*SIN(S1))
      U1=U1+U5*DT(J3)
      U2=U2+U5*DP(J3)
    9 CONTINUE
      U3=U3+U1*C(NS)
      U4=U4+U2*C(NS)
      N3=N3+2
    6 CONTINUE
      S1=.0795774*ETA*BK
      E(1)=-U*S1*U3
      E(2)=-U*S1*U4
      RETURN
      END
```

```
      SUBROUTINE GOMTRY(NBRCH,NEC,NEB,N1,N,CLENTH,BLENTH,THETA)
C
C       THIS SUBROUTINE GENERATING PARAMETERS OF A UMBRELLA SHAPE WIRE
C       STRUCTURE (ANTENNA OR SCATTEREE)
C
C       I/P    NBRCH= NO. OF BRANCHES
C              N1=TOTAL NO. OF SEGMENTS
C              N=TOTAL NUMBER OF EXPANSION FUNCTIONS
C              NEC=NO. OF EXPANSION FUNCTION OVER CENTRAL WIRE
C              NEB=NO. OF EXPANSION FUNCTION OVER EACH BRANCH
C              CLENTH=LENGTH OF CENTRAL WIRE MAY BE NEGATIVE TO REPRE-
C                  SENT THE CASE WITH NEGATIVE-DIRECTED CENTRAL WIRE
C              BLENGTH=LENGTH OF EACH BRANCH (IN METER)
C              THETA=ANGLE BETWEEN EACH BR ANCH AND THE POSITIVE Z-AXIS
C
C       O/P    XX(I) = I'TH COMPONENTS OF THE COORDINATES OF THE CENTER
C                   OF THE I'TH SEGMENT
C              XY(I) IS THE Y COMPONENTS
C              XZ IS THE Z COMPONTS
C              AL(I) IS THE I'TH SEGMENT LENGTH
C              TX IS THE PROJECTTION OF AL TO X-AXIS
C              TY IS THE Y COMPONTS
C              TZ IS THE Z CONPONENT
      COMMON XX(50),XY(50),XZ(50),TX(50),TY(50),TZ(50),AL(50)
      COMMON T(100),TP(100),RAD2(10),L(10),LL(10),LR(10)
      DIMENSION PX(50),PY(50),PZ(50)
      DOUBLE PRECISION DP1,DP2,DP3
      NPB=2*NEB+3
      NPC=2*NEC+3
      DP1=NPC-1
      DP2=CLENTH
      DP2=DP2/DP1
      L(1)=1
      LL(1)=1
      L(2)=1+NEC
      LL(2)=1+NPC
      DO 10 I=1,NPC
      S1=I-1
      PX(I)=0.
      PY(I)=0.
      PZ(I)=S1*DP2
   10 CONTINUE
      DP1=THETA
      DP2=.0174533
      TH=DP1*DP2
      DP1=BLENTH
      DP3=2*NEB

      DP3=DP1/DP3
      D3=DP3*COS(TH)
      D1=DP3*SIN(TH)
      J1=NPC+1
      DO 20 I=1,NBRCH
      BRCH=NBRCH
      X=I-1
```

```
      DP1=360.*X/BRCH
      PH=DP1*DP2
      D2=D1*SIN(PH)
      D4=D1*COS(PH)
      LL(I+2)=LL(I+1)+NPB
      L (I+2)=L(I+1)+NEB
      DO 40 J=1,3
      PX(J1)=0.
      PY(J1)=0.
      PZ(J1)=PZ(4-J)
      J1=J1+1
   40 CONTINUE
      NP1=2*NEB
      DO 30 J=1,NP1
      S1=J
      PX(J1)=D4*S1
      PY(J1)=D2*S1
      PZ(J1)=D3*S1
      J1=J1+1
   30 CONTINUE
   20 CONTINUE
      L(NBRCH+2)=200
      LL(NBRCH+2)=200
      NP=NPC+NBRCH*NPB
      WRITE (3,301)(PX(I),I=1,NP)
      WRITE (3,302) (PY(I),I=1,NP)
      WRITE (3,303)(PZ(I),I=1,NP)
  301 FORMAT('OPX'/(1X, 7E11.4))
  302 FORMAT('OPY'/(1X, 7E11.4))
  303 FORMAT('OPZ'/(1X, 7E11.4))
      J1=1
      J2=2
      N1=0
      DO 2 J=1,NP
      IF (LL(J1)-J) 3,4,3
    4 J2=J2-1
      L(J1)=J2
      J1=J1+1
      GO TO 2
    3 N1=N1+1
      J3=J-1
      IF((N1/2*2-N1).EQ.0)  J2=J2+1
      XX(N1)=.5*(PX(J)+PX(J3))
      XY(N1)=.5*(PY(J)+PY(J3))
      XZ(N1)=.5*(PZ(J)+PZ(J3))
      S1=PX(J)-PX(J3)
      S2=PY(J)-PY(J3)
      S3=PZ(J)-PZ(J3)
      S4=SQRT(S1*S1+S2*S2+S3*S3)
      TX(N1)=S1/S4
      TY(N1)=S2/S4
      TZ(N1)=S3/S4
      AL(N1)=S4
```

```
    2 CONTINUE
      L(J1)=J2
      N=J2-2
      J1=1
      J2=-2
      DO 11 J=1,N
      IF(L(J1)-J) 13,14,13
   14 J2=J2+2
      J1=J1+1
   13 J3=(J-1)*4
      J4=J3+1
      J5=J4+1
      J6=J5+1
      J7=J6+1
      K4=J2+1
      K5=K4+1
      K6=K5+1
      K7=K6+1
      S1=AL(K4)+AL(K5)
      S2=AL(K6)+AL(K7)
      T(J4)=AL(K4)*.5*AL(K4)/S1
      T(J5)=AL(K5)*(AL(K4)+.5*AL(K5))/S1
      T(J6)=AL(K6)*(AL(K7)+.5*AL(K6))/S2
      T(J7)=AL(K7)*.5*AL(K7)/S2
      TP(J4)=AL(K4)/S1
      TP(J5)=AL(K5)/S1
      TP(J6)=-AL(K6)/S2
      TP(J7)=-AL(K7)/S2
      J2=J2+2
   11 CONTINUE
      RETURN
      END


      SUBROUTINE ZBRCH(NBRCH,N1,N,NEC,NEB,Y)
C
C      THIS ROUTINE COMPUTE THE GENERALIZED IMPEDANCE MATRIX OF UMBRELLA
C      SHAPE OF WIRE STRUCTURE
C
C      I/P     NBRCH= NO. OF BRANCHES
C              N1=TOTAL NO. OF SEGMENTS
C              N=TOTAL NUMBER OF EXPANSION FUNCTIONS
C              NEC=NO. OF EXPANSION FUNCTION OVER CENTRAL WIRE
C              NEB=NO. OF EXPANSION FUNCTION OVER EACH BRANCH
C              XX,XY,XZ,AL,TX,TY,TZ ARE GEOMETRY PARAMETERS
C              RAD=RADIUS OF EACH WIRE
C
C      O/P     Y IS THE GENERALIZED IMPEDANCE MATRIX COMPUTED
C
      COMPLEX Z( 400),Y( 400),PSI( 200),U,U1,U2,U3,U4,U5,U6
      COMMON XX(50),XY(50),XZ(50),TX(50),TY(50),TZ(50),AL(50)
      COMMON T(100),TP(100),RAD2(10),L(10),LL(10),LR(10),BK
      DIMENSION DC(200)
      U=(0.,1.)
      PI=3.141593
      ETA=376.7307
```

```
      C1=.125/PI
      C2=.25/PI
      U3=U*BK*ETA
      U4=-U/BK*ETA
      BK2=BK*BK/2.
      BK3=BK2*BK/3.
      N9=0
      N2=1
      NO=1
      N3=-2
      M1=NEC+NFB
      M2=2*M1+4
      M4=NEC
      M3=2*NEC+2
      DO 10 NS=1,M1
      IF(NS.GT.NEC) M3=N1-((NBRCH-1)/2)*(2*NEB+2)
      IF(NS.GT.NEC) M4=NEC+((NBRCH+2)/2)*NEB
      IF(L(N2)-NS) 12,11,12
   11 KK=1
      N3=N3+2
      N2=N2+1
      GO TO 13
   12 KK=3
      DO 14 NF=1,M3
      N4=NF+M3
      N5=N4+M3
      N6=N5+M3
      DC(NF)=DC(N5)
      DC(N4)=DC(N6)
      PSI(NF)=PSI(N5)
      PSI(N4)=PSI(N6)
   14 CONTINUE
   13 IF (N3+1-LR(NO)) 5,6,5
    6 AA=RAD2(NO)
      NO=NO+1
    5 CONTINUE
      DO 15 K=KK,4
      N7=N3+K
      K1=(K-1)*M3
      DO 16 NF=1,M3
      N8=NF+K1
      S1=XX(N7)-XX(NF)
      S2=XY(N7)-XY(NF)
      S3=XZ(N7)-XZ(NF)
      R2=S1*S1+S2*S2+S3*S3+AA
      R=SQRT(R2)
      RT=ABS(S1*TX(N7)+S2*TY(N7)+S3*TZ(N7))
      RT2=RT*RT
      RH=(R2-RT2)
      ALP=.5*AL(N7)
      AR=ALP/R
      S1=BK*R
      U2=COS(S1)-U*SIN(S1)
      IF(AR-.1) 22,22,21
   21 U2=U2*C1/ALP
      S1=RT-ALP
```

```
      S2=RT+ALP
      S3=SQRT(S1*S1+RH)
      S4=SQRT(S2*S2+RH)
      IF(S1) 18,18,19
   18 AI1=ALOG((S2+S4)*(-S1+S3)/RH)
      GO TO 20
   19 AI1=ALOG((S2+S4)/(S1+S3))
   20 AI2=AL(N7)
      AI3=(S2*S4-S1*S3+RH*AI1)/2.
      AI4=AI2*(RH+ALP*ALP/3.+RT2)
      S3=AI1*R
      S1=AI1-BK2*(AI3-R*(2.*AI2-S3))
      S2=-BK*(AI2-S3)+BK3*(AI4-3.*AI3*R+R2*(3.*AI2-S3))
      GO TO 28
   22 U2=U2*C2/R
      BA=BK*ALP
      BA2=BA*BA
      AR2=AR*AR
      AR3=AR2*AR
      ZR=RT/R
      ZR2=ZR*ZR
      ZR3=ZR2*ZR
      ZR4=ZR3*ZR
      H1=(3.-30.*ZR2+35.*ZR4)*AR3/40.
      A1=AR*(-1.+3.*ZR2)/6.+(3.-30.*ZR2+35.*ZR4)*AR3/40.
      A0=1.+AR*A1
      A2=-ZR2/6.-AR2*(1.-12.*ZR2+15.*ZR4)/40.
      A3=AR*(3.*ZR2-5.*ZR4)/60.
      A4=ZR4/120.
      S1=A0+BA2*(A2+BA2*A4)
      S2=BA*(A1+BA2*A3)
   28 PSI(N8)=U2*(S1+U*S2)
      DC(N8)=TX(NF)*TX(N7)+TY(NF)*TY(N7)+TZ(NF)*TZ(N7)
   16 CONTINUE
   15 CONTINUE
      N3=N3+2
      J3=(NS-1)*4
      J7=-2
      J9=1
      DO 25 NF=1,M4
      J1=(NF-1)*4
      IF(L(J9)-NF) 26,27,26
   27 J9=J9+1
      J7=J7+2
   26 N9=N9+1
      U5=0.
      U6=0.
      J5=0
      DO 23 JS=1,4
      J4=J3+JS
      J8=J5+J7
      DO 24 JF=1,4
      J6=J8+JF
      J2=J1+JF
      U5=T(J2)*T(J4)*DC(J6)*PSI(J6)+U5
```

```
      U6=TP(J2)*TP(J4)*PSI(J6)+U6
   24 CONTINUE
      J5=J5+M3
   23 CONTINUE
      Z(N9)=U5*U3+U6*U4
      J7=J7+2
   25 CONTINUE
   10 CONTINUE
      J1=0
      J2=0
      M3=NEC*NEC
      DO 30 I=1,NEC
      J3=M3+I
      DO 31 J=1,NEC
      J2=J2+1
      J1=J1+1
      Y(J1)=Z(J2)
   31 CONTINUE
      S1=NBRCH
      DO 32 J=1,NEB
      J1=J1+1
      Y(J1)=Z(J3)*S1
      J3=J3+M4
   32 CONTINUE
   30 CONTINUE
      J2=M3
      DO 35 I=1,NEB
       DO 36 J=1,NEC
      J2=J2+1
      J1=J1+1
      Y(J1)=Z(J2)
   36 CONTINUE
       DO 37 J=1,NEB
      J1=J1+1
      J2=J2+1
      Y(J1)=Z(J2)
      IF(NBRCH.EQ.1) GO TO 37
      KK=(NBRCH+2)/2
        DO 41 K=2,KK
      KKK=(NBRCH/2)*2-NBRCH
      IF((K.EQ.KK).AND.(0.EQ.KKK)) GO TO 42
      J3=J2+(K-1)*NEB
      Y(J1)=Y(J1)+Z(J3)*2
      GO TO 41
   42 J3=J2+(K-1)*NEB
      Y(J1)=Y(J1)+Z(J3)
   41 CONTINUE
   37 CONTINUE
      J2=J2+(NBRCH/2)*NEB
   35 CONTINUE
      RETURN
      END
```

```
C
C       MAIN PROGRAM           MMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMM
      COMPLEX Z(400),U(20),C(20),E(3),EI(2),UV(4),U1,V
      COMMON XX(50),XY(50),XZ(50),TX(50),TY(50),TZ(50),AL(50)
      COMMON T(100),TP(100),RAD2(10),L(10),LL(10),LR(10),BK
      COMMON /COA/ C
      DIMENSION IFP(20),RAD(10)
    1 READ(1,101,END=500) NB,NEB,NEC,BK,BLENTH,CLENTH,ANGLE
  101 FORMAT(3I3,3E14.7,F6.2)
      WRITE (3,301)NB,NEB,NEC,BK
  301 FORMAT('0 NB NEB NEC      BK'/1X,I3,2I4,E14.7)
      WRITE (3,302) BLENTH,CLENTH,ANGLE
  302 FORMAT( '0LENGTH OF EACH BRANCH '/1X,E14.7// ' LENGTH OF CENTER WI
     1RE'/1X,F14.7//' ANGLE'/1X,F7.2)
      NR=1
      LR(1)=1
      LR(NR+1)=200
      READ(1,109) RAD(1)
  109 FORMAT(E14.7)
      WRITE (3,305) RAD(1)
  305 FORMAT('0RAD'/1X,E14.7)
      DO 46 I=1,NR
      RAD2(I)=RAD(I)**2
   46 CONTINUE
      CALL GOMTRY(NB,NEC,NEB,N1,N,CLENTH,BLENTH,ANGLE)
      CALL ZBRCH(NB,N1,N,NEC,NEB,Z)
      NE=NEC+NEB
      NE2=NE*NE
      CALL LINEQ (NE,Z)
      DO 45 I=1,N
      U(I)=0.
   45 CONTINUE
      READ (1,107) NF
  107 FORMAT(20I3)
      WRITE (3,303)NF
  303 FORMAT ('0NF'/I3/'0FP                    VOLTAGE')
      DO 44 I=1,NF
      READ (1,119) J1,V
  119 FORMAT(I3,2E14.7)
      WRITE (3,120) J1,V
  120 FORMAT(1X,I3,2E14.7)
      U(J1)=V
      IFP(I)=J1
   44 CONTINUE
      WRITE (3,111)
  111 FORMAT ('0CURRENT'/'     I      REAL            IMAG              MAGNITUDE
     1     PHASE')
      J3=0
      J1=0
    5 J1=J1+1
      U1=0.
      J2=0
    6 J2=J2+1
      J3=J3+1
      U1=U1+Z(J3)*U(J2)
      IF (J2-NE)6,7,7
    7 C(J1)=U1
      IF (J1-NE)5,8,8
    8 CONTINUE
```

```
      DO 9 I=1,NE
      U1=C(I)
      CM=CABS(U1)
      IF (CM) 11,12,11
   11 CP=ATAN2(AIMAG(U1),REAL(U1))*57.2858
      GO TO 10
   12 CP=0.
   10 CONTINUE
      WRITE (3,112) I,C(I),CM,CP
  112 FORMAT (I5,3E14.6,F10.3)
    9 CONTINUE
      IF(NB.EQ.1) GO TO 42
      J1=NEB+NEC
      DO 41 I=2,NB
      J2=NEB
      DO 41 J=1,NEB
      J1=J1+1
      J2=J2+1
      C(J1)=C(J2)
   41 CONTINUE
   42 CONTINUE
      WRITE (3,113)
  113 FORMAT ('OFIELD PATTERN'/'   K=1 FOR THETA COMPONENT   K=2 FOR PHI

     1 COMPONENT'/'0 THETA PHI  K       REAL          IMAG         MAGNITUD
     2E      PHASE')
      DO 17 IPH=1,2
      PHI=(IPH-1)*90
      PH=PHI*.0174533
      DO 17 ITH=1,181,5
      THE=ITH-1
      TH=THE*.0174533
      CALL ROW (N,TH,PH,E)
      DO 16 K=1,2
      U1=E(K)
      GM=CABS(U1)
      IF (GM)13,13,14
   14 EP=ATAN2 (AIMAG(U1),REAL(U1))
      GO TO 15
   13 EP=0.
   15 GP=EP*57.2858
      WRITE (3,114) THE,PHI,K,U1,GM,GP
  114 FORMAT(F6.0,F5.0,I3,3E14.7,F10.3)
   16 CONTINUE
   17 CONTINUE
      GO TO 1
  500 STOP
      END
/*
//GO.SYSIN DD *
  2  4  6+0.6283185E+01+0.1250000E+00+0.2500000E+00+90.00
+0.1000000E-03
  1
  7+0.1000000E+01+0.0000000E+00
```

```
  NB NEB NFC      BK
  2   4   6 0.6283184E 01

LENGTH OF EACH BRANCH
 0.1250000E 00

LENGTH OF CENTER WIRE
 0.2500000E 00

ANGLE
  90.00

RAD
 0.9999999E-04

PX
 0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.1563E-01 0.3125E-01 0.4688E-01
 0.6250E-01 0.7813E-01 0.9375E-01 0.1094E 00 0.1250E 00 0.0        0.0
 0.0       -0.1563E-01-0.3125E-01-0.4688E-01-0.6250E-01-0.7813E-01-0.9375E-01
-0.1094E 00-0.1250E 00

PY
 0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0        0.0        0.0        0.0        0.0        0.0        0.0
 0.0       -0.5089E-08-0.1018E-07-0.1527E-07-0.2036E-07-0.2544E-07-0.3053E-07
-0.3562E-07-0.4071E-07

PZ
 0.0        0.1786E-01 0.3571E-01 0.5357E-01 0.7143E-01 0.8929E-01 0.1071E 00
 0.1250E 00 0.1429E 00 0.1607E 00 0.1786E 00 0.1964E 00 0.2143E 00 0.2321E 00
 0.2500E 00 0.3571E-01 0.1786E-01 0.0        0.4906E-08 0.9812E-08 0.1472E-07
 0.1962E-07 0.2453E-07 0.2944E-07 0.3434E-07 0.3925E-07 0.3571E-01 0.1786E-01
 0.0        0.4906E-08 0.9812E-08 0.1472E-07 0.1962E-07 0.2453E-07 0.2944E-07
 0.3434E-07 0.3925E-07

NF
 1

FP             VOLTAGE
  7 0.1000000E 01 0.0

CURRENT
    I       REAL           IMAG          MAGNITUDE      PHASE
    1 -0.511069E-03 -0.476490E-02  0.479223E-02   -96.105
    2 -0.508337E-03 -0.437440E-02  0.440384E-02   -96.612
    3 -0.471473E-03 -0.380923E-02  0.383829E-02   -97.039
    4 -0.400856E-03 -0.307407E-02  0.310009E-02   -97.412
    5 -0.298497E-03 -0.218866E-02  0.220892E-02   -97.749
    6 -0.167006E-03 -0.117641E-02  0.118820E-02   -98.063
    7  0.241828E-03  0.253185E-02  0.254337E-02    84.529
    8  0.195378E-03  0.199148E-02  0.200105E-02    84.382
    9  0.138629E-03  0.139732E-02  0.140418E-02    84.319
   10  0.745126E-04  0.744863E-03  0.748581E-03    84.273
```

FIELD PATTERN

K=1 FOR THETA COMPONENT   K=2 FOR PHI COMPONENT

| THETA | PHI | K | REAL | IMAG | MAGNITUE | PHASE |
|---|---|---|---|---|---|---|
| 0. | 0. | 1 | 0.1852448E-01 | -0.2404486E-02 | 0.1867987E-01 | -7.394 |
| 0. | 0. | 2 | -0.4318437E-08 | 0.2318754E-09 | 0.4324658E-08 | 176.896 |
| 5. | 0. | 1 | 0.2811516E-01 | 0.4627321E-02 | 0.2849341E-01 | 9.345 |
| 5. | 0. | 2 | -0.4283006E-08 | 0.5410978E-09 | 0.4317048E-08 | 172.770 |
| 10. | 0. | 1 | 0.3760987E-01 | 0.1145996E-01 | 0.3931709E-01 | 16.943 |
| 10. | 0. | 2 | -0.4232792E-08 | 0.8457961E-09 | 0.4316465E-08 | 168.671 |
| 15. | 0. | 1 | 0.4702598E-01 | 0.1788098E-01 | 0.5031075E-01 | 20.815 |
| 15. | 0. | 2 | -0.4168804E-08 | 0.1143016E-08 | 0.4322661E-08 | 164.639 |
| 20. | 0. | 1 | 0.5636786E-01 | 0.2368479E-01 | 0.6114167E-01 | 22.787 |
| 20. | 0. | 2 | -0.4092520E-08 | 0.1429995E-08 | 0.4335156E-08 | 160.712 |
| 25. | 0. | 1 | 0.6562126E-01 | 0.2867664E-01 | 0.7161349E-01 | 23.601 |
| 25. | 0. | 2 | -0.4005798E-08 | 0.1704222E-08 | 0.4353250E-08 | 156.926 |
| 30. | 0. | 1 | 0.7474923E-01 | 0.3267739E-01 | 0.8157974E-01 | 23.609 |
| 30. | 0. | 2 | -0.3910849E-08 | 0.1963477E-08 | 0.4376066E-08 | 153.314 |
| 35. | 0. | 1 | 0.8368844E-01 | 0.3552905E-01 | 0.9091789E-01 | 22.999 |
| 35. | 0. | 2 | -0.3810115E-08 | 0.2205868E-08 | 0.4402590E-08 | 149.905 |
| 40. | 0. | 1 | 0.9234822E-01 | 0.3710096E-01 | 0.9952223E-01 | 21.884 |
| 40. | 0. | 2 | -0.3706218E-08 | 0.2429835E-08 | 0.4431719E-08 | 146.725 |
| 45. | 0. | 1 | 0.1006104E 00 | 0.3729643E-01 | 0.1073008E 00 | 20.336 |
| 45. | 0. | 2 | -0.3601869E-08 | 0.2634155E-08 | 0.4462311E-08 | 143.796 |
| 50. | 0. | 1 | 0.1083333E 00 | 0.3605935E-01 | 0.1141769E 00 | 18.407 |
| 50. | 0. | 2 | -0.3499781E-08 | 0.2817917E-08 | 0.4493231E-08 | 141.135 |
| 55. | 0. | 1 | 0.1153556E 00 | 0.3338032E-01 | 0.1200880E 00 | 16.136 |
| 55. | 0. | 2 | -0.3402594E-08 | 0.2980492E-08 | 0.4523379E-08 | 138.759 |
| 60. | 0. | 1 | 0.1215046E 00 | 0.2930132E-01 | 0.1249877E 00 | 13.556 |
| 60. | 0. | 2 | -0.3312790E-08 | 0.3121491E-08 | 0.4551733E-08 | 136.679 |
| 65. | 0. | 1 | 0.1266061E 00 | 0.2391834E-01 | 0.1288456E 00 | 10.696 |
| 65. | 0. | 2 | -0.3232655E-08 | 0.3240725E-08 | 0.4577373E-08 | 134.905 |
| 70. | 0. | 1 | 0.1304952E 00 | 0.1738180E-01 | 0.1316477E 00 | 7.586 |
| 70. | 0. | 2 | -0.3164200E-08 | 0.3338133E-08 | 0.4599485E-08 | 133.444 |
| 75. | 0. | 1 | 0.1330277E 00 | 0.9892862E-02 | 0.1333951E 00 | 4.252 |
| 75. | 0. | 2 | -0.3109131E-08 | 0.3413756E-08 | 0.4617405E-08 | 132.303 |
| 80. | 0. | 1 | 0.1340917E 00 | 0.1697588E-02 | 0.1341023E 00 | 0.725 |
| 80. | 0. | 2 | -0.3068799E-08 | 0.3467675E-08 | 0.4630579E-08 | 131.485 |
| 85. | 0. | 1 | 0.1336166E 00 | -0.6923538E-02 | 0.1337957E 00 | -2.966 |
| 85. | 0. | 2 | -0.3044198E-08 | 0.3499985E-08 | 0.4638643E-08 | 130.993 |
| 90. | 0. | 1 | 0.1315810E 00 | -0.1566721E-01 | 0.1325104E 00 | -6.789 |
| 90. | 0. | 2 | -0.3035932E-08 | 0.3510747E-08 | 0.4641358E-08 | 130.829 |
| 95. | 0. | 1 | 0.1280149E 00 | -0.2422226E-01 | 0.1302862E 00 | -10.713 |
| 95. | 0. | 2 | -0.3044198E-08 | 0.3499985E-08 | 0.4638643E-08 | 130.993 |
| 100. | 0. | 1 | 0.1230005E 00 | -0.3228397E-01 | 0.1271666E 00 | -14.704 |
| 100. | 0. | 2 | -0.3068799E-08 | 0.3467677E-08 | 0.4630582E-08 | 131.485 |
| 105. | 0. | 1 | 0.1166678E 00 | -0.3956900E-01 | 0.1231953E 00 | -18.732 |
| 105. | 0. | 2 | -0.3109131E-08 | 0.3413756E-08 | 0.4617405E-08 | 132.303 |
| 110. | 0. | 1 | 0.1091857E 00 | -0.4582763E-01 | 0.1184132E 00 | -22.765 |
| 110. | 0. | 2 | -0.3164200E-08 | 0.3338133E-08 | 0.4599485E-08 | 133.444 |
| 115. | 0. | 1 | 0.1007528E 00 | -0.5085400E-01 | 0.1128594E 00 | -26.777 |
| 115. | 0. | 2 | -0.3232657E-08 | 0.3240725E-08 | 0.4577373E-08 | 134.905 |
| 120. | 0. | 1 | 0.9158385E-01 | -0.5449336E-01 | 0.1065698E 00 | -30.748 |
| 120. | 0. | 2 | -0.3312790E-08 | 0.3121491E-08 | 0.4551733E-08 | 136.679 |
| 125. | 0. | 1 | 0.8189815E-01 | -0.5664512E-01 | 0.9957898E-01 | -34.664 |
| 125. | 0. | 2 | -0.3402594E-08 | 0.2980492E-08 | 0.4523379E-08 | 138.759 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 170. | 0. | 1 | -0.2737458E-02 | -0.1548368E-01 | 0.1572380E-01 | -100.009 |
| 170. | 0. | 2 | -0.4232792E-08 | 0.8457970E-09 | 0.4316465E-08 | 168.671 |
| 175. | 0. | 1 | -0.1072789E-01 | -0.6631736E-02 | 0.1261220E-01 | -148.251 |
| 175. | 0. | 2 | -0.4283006E-08 | 0.5410976E-09 | 0.4317048E-08 | 172.770 |
| 180. | 0. | 1 | -0.1852450E-01 | 0.2404514E-02 | 0.1867989E-01 | 172.574 |
| 180. | 0. | 2 | -0.4318437E-08 | 0.2318762E-09 | 0.4324658E-08 | 176.896 |
| 0. | 90. | 1 | 0.1497844E-08 | -0.5230831E-09 | 0.1586554E-08 | -19.247 |
| 0. | 90. | 2 | -0.1852448E-01 | 0.2404486E-02 | 0.1867987E-01 | 172.574 |
| 5. | 90. | 1 | 0.9491581E-02 | 0.5323514E-02 | 0.1088255E-01 | 29.282 |
| 5. | 90. | 2 | -0.1852448E-01 | 0.2404486E-02 | 0.1867987E-01 | 172.574 |
| 10. | 90. | 1 | 0.1901302E-01 | 0.1048635E-01 | 0.2171309E-01 | 28.873 |
| 10. | 90. | 2 | -0.1852448E-01 | 0.2404485E-02 | 0.1867987E-01 | 172.574 |
| 15. | 90. | 1 | 0.2858743E-01 | 0.1532894E-01 | 0.3243791E-01 | 28.196 |
| 15. | 90. | 2 | -0.1852448E-01 | 0.2404482E-02 | 0.1867987E-01 | 172.574 |
| 20. | 90. | 1 | 0.3822480E-01 | 0.1969456E-01 | 0.4300012E-01 | 27.254 |
| 20. | 90. | 2 | -0.1852448E-01 | 0.2404482E-02 | 0.1867987E-01 | 172.574 |
| 25. | 90. | 1 | 0.4791601E-01 | 0.2343156E-01 | 0.5333837E-01 | 26.055 |
| 25. | 90. | 2 | -0.1852448E-01 | 0.2404481E-02 | 0.1867987E-01 | 172.574 |
| 30. | 90. | 1 | 0.5762853E-01 | 0.2639677E-01 | 0.6338638E-01 | 24.606 |
| 30. | 90. | 2 | -0.1852448E-01 | 0.2404481E-02 | 0.1867987E-01 | 172.574 |
| 35. | 90. | 1 | 0.6730169E-01 | 0.2845993E-01 | 0.7307178E-01 | 22.918 |
| 35. | 90. | 2 | -0.1852448E-01 | 0.2404480E-02 | 0.1867987E-01 | 172.574 |
| 40. | 90. | 1 | 0.7684582E-01 | 0.2950929E-01 | 0.8231694E-01 | 21.003 |
| 40. | 90. | 2 | -0.1852448E-01 | 0.2404480E-02 | 0.1867987E-01 | 172.574 |
| 45. | 90. | 1 | 0.8614147E-01 | 0.2945792E-01 | 0.9103906E-01 | 18.876 |
| 45. | 90. | 2 | -0.1852448E-01 | 0.2404479E-02 | 0.1867987E-01 | 172.574 |
| 50. | 90. | 1 | 0.9504282E-01 | 0.2825044E-01 | 0.9915251E-01 | 16.551 |
| 50. | 90. | 2 | -0.1852448E-01 | 0.2404479E-02 | 0.1867987E-01 | 172.574 |
| 55. | 90. | 1 | 0.1033820E 00 | 0.2586960E-01 | 0.1065695E 00 | 14.046 |
| 55. | 90. | 2 | -0.1852448E-01 | 0.2404476E-02 | 0.1867988E-01 | 172.574 |
| 60. | 90. | 1 | 0.1109769E 00 | 0.2234155E-01 | 0.1132034E 00 | 11.380 |
| 60. | 90. | 2 | -0.1852448E-01 | 0.2404476E-02 | 0.1867988E-01 | 172.574 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 150. | 90. | 1 | 0.4938317E-01 | -0.3884482E-01 | 0.6283003E-01 | -38.182 |
| 150. | 90. | 2 | -0.1852448E-01 | 0.2404480E-02 | 0.1867988E-01 | 172.574 |
| 155. | 90. | 1 | 0.4069822E-01 | -0.3371320E-01 | 0.5284812E-01 | -39.630 |
| 155. | 90. | 2 | -0.1852448E-01 | 0.2404480E-02 | 0.1867988E-01 | 172.574 |
| 160. | 90. | 1 | 0.3222238E-01 | -0.2784979E-01 | 0.4258981E-01 | -40.830 |
| 160. | 90. | 2 | -0.1852448E-01 | 0.2404480E-02 | 0.1867988E-01 | 172.574 |
| 165. | 90. | 1 | 0.2395230E-01 | -0.2139971E-01 | 0.3211947E-01 | -41.771 |
| 165. | 90. | 2 | -0.1852448E-01 | 0.2404481E-02 | 0.1867988E-01 | 172.574 |
| 170. | 90. | 1 | 0.1585945E-01 | -0.1451008E-01 | 0.2149569E-01 | -42.449 |
| 170. | 90. | 2 | -0.1852448E-01 | 0.2404481E-02 | 0.1867988E-01 | 172.574 |
| 175. | 90. | 1 | 0.7895697E-02 | -0.7327948E-02 | 0.1077222E-01 | -42.857 |
| 175. | 90. | 2 | -0.1852448E-01 | 0.2404481E-02 | 0.1867988E-01 | 172.574 |
| 180. | 90. | 1 | -0.3092213E-07 | 0.2796228E-07 | 0.4169014E-07 | 137.854 |
| 180. | 90. | 2 | -0.1852448E-01 | 0.2404484E-02 | 0.1867988E-01 | 172.574 |